CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUATION (ARTS, SCIENCE), BACHELOR OF SCIENCE

MATH 400: TOPOLOGY 1

STREAMS: **TIME: 2 HOURS**

DAY/DATE: FRIDAY 8/12/2017 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS:

- **Answer question one and any other TWO questions**
- **Do not write on the question paper**

QUESTION ONE: (30 MARKS)

(a) Distinguish the following terms as used in topology

(i) An indiscrete topology and Sierpinski topology

(ii) A base for the topology τ and a local basis at the point p

(iii) A $T_1 and\ T_2$ space

(iv) A regular space and a normal space (8marks)

(b) (i) Let $X = \{a, b, c, d, e\}$. Determine whether or not the class

$\tau_A = \{\{a, b, c\}, \{a, b, c, d\}, \{a, b, d\}, X, \emptyset\}$of subsets of X is a topology on X. (2marks)

(ii) Let τ be a topology on a set X consisting of four sets i.e. $\tau = \{X, \emptyset, A, B\}$. What conditions must A and B satisfy? (2marks)

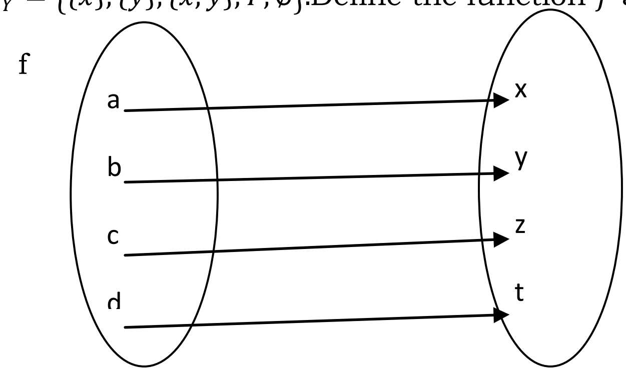
(c) Prove that if A is a subset of a discrete topology, then set of its derived points A' is empty (4marks)

(d) Consider the following topology on $X = \{a, b, c, d, e\}$ and

$\tau = \{\{a\}, \{a, b\}, \{a, c, d\}, \{a, b, e\}, \{a, b, c, d\}, X, \emptyset\}$.

Given the sets $A = \{a\}$, $B = \{b\}$, $C = \{c, e\}$, which ones are dense in X? (6marks)

(e) Let $X = \{a, b, c, d\}$ $with$ $\tau_X = \{\{a, b\}, \{a\}, \{b\}, X, \emptyset\}$ and Let $Y = \{x, y, z, t\}$ $with$ $\tau_Y = \{\{x\}, \{y\}, \{x, y\}, Y, \emptyset\}$.Define the function f as



Show that the function f is a homomorphism. (4marks)

(f) Let $f: X \to Y$ be a constant function. Prove that then f is continuous relative to τ_X and τ_Y. (4marks)

QUESTION TWO: (20 MARKS)

(a) Prove that a topological space X is a T_1 space iff every singleton subset $\{p\} \subset X$ is closed. (5marks)

(b) Let A be a subset of a topological space(X, τ). Prove that τ_A is a topology on A, where $\tau_A = \{A \cap G: G \in \tau\}$ (6marks)

(c) Let $p \in X$ and denote N_P the set of all neighborhood of a point p. Prove that

(i) $N_P \neq \emptyset \forall \; N \in N_P, \; p \in N$

(ii) $\forall \; pairs \; N, M \in N_P, \; N \cap M \in N_P$

(iii) If $N \in N_P$ and for every $M \subset X$ with $N \subset M$ it implies that $M \in N_P$

(5marks)

(d) Let $f: x_1 \to x_2 where x_1 = x_2 = \{0,1\}$ and are such that $(x_1, D) and \; (x_2, \$)$ be defined by $f(1) = 1 \;\; and f(0) = 0$. Show that f is continuous whereas f^{-1} is not. (4marks)

QUESTION THREE: (20 MARKS)

(a) Consider the following topology on $X = \{a, b, c, d, e\}$ and

$\tau = \{\{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}, X, \emptyset\}$. If $A = \{a, b, c\}$. Find

(i) The exterior of A (3marks)

(ii) The boundary of A (2marks)

(iii) Hence show that the boundary of A, $\delta A = \bar{A} \cap \overline{X/A}$ (3marks)

(b) Let $A \subset X$, where X is a non-empty topological space. Prove that$\bar{A} = \delta A \; \cup A^0$ (7marks)

(c) If θ is a subbasis for the topologies τ and τ^*on X, show that $\tau = \tau^*$ (5marks)

QUESTION FOUR: (20 MARKS)

(a) (i) Let $f: X \to Y$ be any function and (Y, τ_Y)an indiscrete topological space. Show that f is continuous (2marks)

(ii) Let $f: X \to Y$ and $g: Y \to Z$ be continuous functions. Prove that the composite function $g \circ f$ is continuous (5marks)

(b) Let $f: X \to Y$ be a bijective. Prove that the following statements are equivalent.

(i) f is a homomorphism

(ii) f is open

(iii) f is closed

(iv) $f(\bar{A}) = \overline{\overline{f(A)}}$ (13marks)

QUESTION FIVE: (20 MARKS)

(a) Using an appropriate counter examples show that a T_2space $\Rightarrow T_1$space and T_1 space $\Rightarrow T_0$space but a T_0 space $\nRightarrow T_1$ and a T_1 space $\nRightarrow T_2$. (8marks)

(b) Prove that a topological space X is a T_1 space iff every singleton subset$\{p\} \subset X$ is closed. (6marks)

(c) Let $P: X \to Y$ be an open map and let $S \subset Y$ be any subset of Y and A is a closed set in X such that $P^{-1}(S) \subset A$. Show that $S \subset B$ and $P^{-1}(B) \subset A$. (6marks)
