CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DEGREE OF BACHELOR OF EDUCATION (ARTS) ,BACHELOR OF ARTS (MATHS- ECONS)

MATH 220: VECTOR ANALYSIS

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 14/12/2017 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Distinguish the following terms as used in vector analysis:

(i) Linearly dependent vectors and Linearly independence vectors (2 marks)

(ii) The dot product and cross product of two vectors $\vec{A}$ and $\vec{B}$ (2 marks)

(iii) The gradient of a scalar function $\emptyset$ and the divergence of the vetor$\vec{V}$ (2 marks)

(iv) An irrotational vector and a solenoidal vector $\vec{V}$ (2 marks)

(v) Rectangular coordinates and curvilinear coordinates of a point P (2 marks)

(b) (i) Show that addition of two vectors is commutative (2 marks)

(ii) Prove that if $\vec{a}$ and $\vec{b}$ are non-collinear vectors then $x\vec{a} + y\vec{b} = 0 \ implies \ x = y = 0$ (2marks)

(c) Given $\vec{A} = A_1\hat{\imath} + A_2\hat{\jmath} + A_3\hat{k}$ and $\vec{B} = B_1\hat{\imath} + B_2\hat{\jmath} + B_3\hat{k}$, show that

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$ (3 marks)

(d) Using vector method find the area of the triangle having vertices $P(2,3,5), Q(4,2,-1), R(3,6,4)$ (4 marks)

(e) If $\vec{A} = (2x^2y - x^4)\hat{\imath} + (e^{xy} - ysinx)\hat{\jmath} + (x^2cosy)\hat{k}$, find $\frac{\delta^2 A}{\delta x \delta y}$ (3 marks)

(f) Find the work done in moving a particle in a force field given by $\vec{F} = (2xy)\hat{\imath} - (5z)\hat{\jmath} + (10x)\hat{k}$ along $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$ (4 marks)

(g) State without proof the Green's theorem in a plane (2 marks)

QUESTION TWO: (20 MARKS)

(a) Find the equation for the plane perpendicular to the vector $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ and passing through the terminal point of the vector $B = \hat{\imath} + 5\hat{\jmath} + 3\hat{k}$ (3 marks)

(b) (i) Show that $\nabla \cdot \nabla\emptyset = \nabla^2\emptyset$, where $\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$ denotes the laplacian operator.(4 marks)

(ii) Hence show that $\nabla^2\left(\frac{1}{r}\right) = 0$ (8 marks)

(c) Given that $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$. Evaluate $\int_C \vec{A} \cdot d\vec{r}$ between the points (0,0,0) and (1,1,0) (5 marks)

QUESTION THREE: (20 MARKS)

(a) (i)Given that $\vec{A} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\widehat{k}$ .Find the constants a, b *and* c such that the vector $\vec{A}$ is irrotational. (3marks)

(ii) Hence show that the vector $\vec{A}$ in (c,(i)) can be expressed in as a gradient of a scalar function $\emptyset$ (10 marks)

(b) State without proof the Stoke's theorem. Hence evaluate the surface integral of the normal component of a vector $\vec{A} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} + y^2z\,\hat{k}$ taken over the surface S of the upper half of the sphere

$x^2 + y^2 + z^2 = 1$ (7 marks)

QUESTION FOUR: (20 MARKS)

(a) Given that $\emptyset = 45x^2y$, evaluate its volume integral such that the volume space is bounded by planes $4x + 2y + z = 8,\ x = 0, y = 0, z = 0.$ (6 marks)

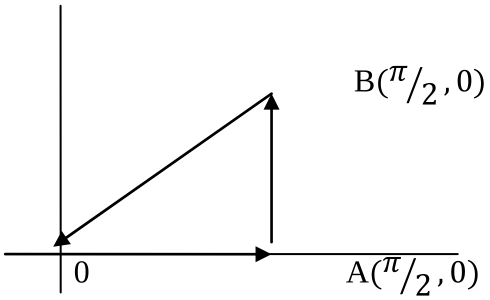
(b)(i) Verify the Green's Theorem for $\int_C (xy + y^2)dx + x^2dy$,

$where\ C\ is\ the\ closed\ curve\ of\ the\ region\ bounded\ by\ y = x\ and\ y = x^2$ (8marks)

(ii) Using Green's Theorem

evalauate $\int_C (y - sinx)dx + cosxdy\ , where\ C\ is\ the\ trainlge\ of\ the\ adjoining\ figure$:

(6 marks)



QUESTION FIVE: (20 MARKS)

(a) State without proofthe Frenet-Serret formulas (3 marks)

(b) Given the space curve defined by $x = 3cost, y = 3sint, z = 4t$.Find

(i) The tangent vector $\vec{T}$ (3 marks)

(ii) The principal normal $\vec{N}$ (3 marks)

(iii) The Binormal$\vec{B}$ (2 marks)

(c) Given$\vec{A} = 18z\,\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$. Evaluate $\iint_S \vec{A} \cdot d\vec{s}$ where the surface S is the part of the plane $2x + 3y + 6z = 12$ in the first octant. (9 marks)
