

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DEGREE OF BACHELOR OF EDUCATION (ARTS,SCIENCE) ,BACHELOR OF SCIENCE, BACHELOR OF ARTS (MATHS-ECON), BACHELOR OF SCIENCE (COMP SCI,APPLIED COMP SCI),BACHELOR OF SCIENCE (ECON STATS)

MATH 122: BASIC MATHEMATICS

STREAMS:

DAY/DATE: WEDNESDAY 13/12/2017

TIME: 2 HOURS

11.30 A.M – 1.30 P.M

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

Do not write on the question paper.

QUESTION ONE (30 MARKS)

- a. Using venn diagrams show that (i) $A - (B \cup C) = (A-B) \cap (A-C)$ [2marks]
(ii) $(A-B) \cap (A \cap B) = \emptyset$ (2 marks)
- b. (i) Let r denote "It is raining" and s denote "it is snowing". Write the English sentences corresponding to the following ;
- $r \wedge \sim s$ (1 mark)
 - $\sim r \rightarrow s$ (1 mark)
 - $\sim r \leftrightarrow \sim(s \wedge r)$ (1 mark)
- c. Construct the truth table for the following proposition to determine whether is a fallacy, tautology or an in derterminate.

$$(p \rightarrow q) \leftrightarrow [\sim q \rightarrow (\sim p \wedge \sim q)] \quad (3marks)$$

- d. Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 2, 3\}$. Define a relation R by the set

$$R = \{(x, y) \text{ such that } x \leq y \quad \forall x \in X, y \in Y\}. \text{ Find the set } R. \quad (3marks)$$

- e. How many ways are there to select a first prize winner, a second prize winner and a third prize winner from 50 different people who have entered a contest? (3 marks)

- f. Let z_1 and z_2 be two complex numbers. Prove that $|z_1 z_2| = |z_1| |z_2|$ (3 marks)
- g. Given the functions $f(x) = 2x - 1$ and $g(x) = \frac{1}{3}x^2$, show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$. (5 marks)
- h. (i) Evaluate $\sum_{n=1}^{11} \left(\frac{3}{4}\right)^n$
 (3marks)(ii) Solve for p in the equation $p_{C_2} = 28$. (3marks)

QUESTION TWO 20 MARKS)

- a. A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways can they be seated if
 i. All the sisters sit together.
 ii. No two sisters sit together. (4 marks).
- b. (i) Given the complex numbers $z_1 = -5 - 3i$ and $z_2 = 3 - 4i$, Express $\frac{z_1 - z_2}{z_1 z_2}$ in form of $a + bi$ (4 marks)
- (ii) Let $z_1 = (7, 42^0)$, $z_2 = (2, 23^0)$, show that $z_1 z_2 = (14, 65^0)$ (4marks)
- c. State DeMoivre's theorem and use it to show that $(\cos \theta + i \sin \theta)^{k+1} = \cos(k\theta + \theta) + i \sin(k\theta + \theta)$ (4 marks)
- d. Given the functions $r: x \rightarrow \frac{1}{2}x + 4$ and $r \circ s: x \rightarrow \frac{9x - 1}{2x}$, find $s(x)$. (4 marks)

QUESTION THREE (20 MARKS)

- a. Prove that $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n . (5 marks)
- b. Market segmentation is known as the process of dividing up a market for goods or services into smaller groups e.g. geographic, demographics and psychographics. A total of 1800 advertisements were run to try and reach various segments. The following list shows the breakdown of each :
- 680 for geographic location
 - 500 based on demographics
 - 440 based on psychographics
 - 385 based on both demographics and geographic
 - 245 based on psychographics and geographic
 - 325 based on both demographics and psychographic

245 based on all the three segments.

Using a Venn diagram, find the number which were:

- i. Run either for geographic or demographics (4 marks)
 - ii. Not run for geographic (2 marks)
 - iii. Not run for any of the these segments (2 marks)
 - iv. Not run for geographic but were run demographics (2 marks)
- c. Use the analytic method to prove $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ (5marks)

QUESTION FOUR (20 MARKS)

- (a) (i) Prove that $\cos(30^\circ + x) = \frac{\sqrt{3} \cos x - \sin x}{2}$ (3marks)
- (ii) Solve the equation $\sin \theta - 3 \cos \theta = 2$ for $0 \leq \theta \leq 360^\circ$. (5 marks)
- (b) Expand $\frac{1}{(1+x)^2}$ up to the term containing x^3 (4marks)
- (c) Determine the domain and the range of the function; $y = \sqrt{x^2 - 4}$. (3 marks)
- (d) If $\sin(x + \alpha) = 2 \cos(x - \alpha)$, prove that $\tan x = \frac{2 - \tan \alpha}{1 - 2 \tan \alpha}$ (5 marks)

QUESTION FIVE (20 MARKS)

- a. Prove that $\sqrt{8}$ is not a rational number. (4 marks)
- b. Use mathematical induction to prove that for any positive integer n , $n^3 + 2n$ is divisible by 3. (6marks)
- c. Evaluate $(1 + i)^{11}$ giving your answer in the form $a + ib$. (5 marks)
- d. In the Venn diagram below, the number of elements are shown. Given that $\varepsilon = A \cup B \cup C$ and $n(\varepsilon) = 35$
 Find (i) $n(A \cup C)$ (2 marks)
 (ii) $n(A \cap C)$ (1 marks)
 (iii) $n(A' \cap C')$ (2 marks)