CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE, BACHELOR OF EDUCATION ARTS, BACELOR OF SCIENCE GENERAL, BACHELOR OF SCIENCE ECONOMICS AND STATISTICS AND BACHELOR OF ARTS ECONOMICS AND MATHEMATICS

MATH 452: HYPOTHESIS TESTING

STREAMS: B.Ed (SCI), BED (ARTS) B.Sc (GEN) B.Sc (ECONSTAT) & ECONMATH

TIME: 2 HOURS

DAY/DATE: FRIDAY 8/12/2017 **11.30 A.M - 1.30 P.M.**

INSTRUCTIONS:

- Answer Question ONE and any other TWO Questions.
- All workings must be shown clearly

QUESTION ONE [30 MARKS]

(a) Define the following terms as used in hypothesis testing:
   (i) A statistical test
   (ii) Randomized test
   (iii)Power of a test
   (iv)Composite hypothesis
   (v) Uniformly most powerful test [5 Marks]

(b) Let X represent a single observation from the p.d.f given by:

$$f(x;d) = \begin{cases} \delta x^{\delta-1}, & 0 < x < 1 \\ 0, & \text{Otherwise} \end{cases}$$

Find the Most powerful size $a = 0.01$ test to test $H_0: \delta = 1$ against $H_1: \delta = \frac{1}{4}$ [6 Marks]

(c) Jacobian performed an experiment four times where the probability of success was β. He set the hypothesis as: $H_0: \beta = \frac{2}{5}$ against $H_1: \beta = 0.25$. The null hypothesis was rejected whenever the number of successes was less than two. Determine:
   (i) The probability of type 1 error
   (ii) Probability of type II error

(iii)The power of the test when $\beta = \frac{1}{3}$. [7 Marks]

(d) State the four elements of a statistical test. [2 Marks]

(e) Define the generalized likelihood ratio test and state four conditions satisfied by λ_n.[6 Marks]
(f) Explain the theory of sequential likelihood ratio testing and its goals. [4 Marks]

QUESTION TWO

(a) Let X be a random variable from a poisson distribution with parameter μ. Suppose the test is set at $H_0: \mu = 2$ against $H_1: \mu \neq 2$. Find the generalized likelihood test give that $a = 0.05$.
(b) Suppose that $y_{1,} y_{2,\ldots\ldots\ldots} y_n$ is a random sample from a normal distribution with an unknown mean of μ and a variance of σ^2. Find the appropriate likelihood ratio test. [10 Marks]

QUESTION THREE

(a) Suppose that $x_{1,} x_{2,} \ldots \ldots \ldots . x_n$ is a random sample of size 50 from a normal population whose mean μ is unknown and standard deviation is 1. Consider the hypothesis:$H_0: \mu = 4$ against $H_1: \mu = 6$.
   (i) Construct a most powerful size $a = 0.01$test.
   (ii) Determine the value of k for the critical region. [10 Marks]

(b) Let X be a random variable with a random sample size $n = 20$ from the Bernoulli distribution.

$$f(x;\theta) = \begin{cases} \theta^x (1-\theta)1-x, & \text{x = 0, 1} \\ 0, & \text{elsewhere} \end{cases}$$

Obtain the Most Powerful size $a = 0.05$ test for testing $H_0: \theta = 0.02$ against$H_1: \theta = 0.04$.
[10 Marks]

QUESTION FOUR

An experiment was set to test the effectiveness of a drug to disease. The drug dosage and reaction time to disease symptoms were recommended as below for each of the subjects receiving the drug.

Dosage (g) X	8	6	4	6	10	4	8	10
Reaction Time (seconds) Y	3.9	4.0	7.5	4.4	1.4	6.8	3.1	1.7

Test the hypothesis:
H_0:The linear relationship between drug dosage and reaction time is not significant.
H_1: The linear relationship between drug dosage and reaction time is significant. [8 Marks]

QUESTION FIVE

(a) A Company produces two products and wishes to compare the popularity of the two products. 100 independent observations on the number of sales gave them the mean $\bar{X} = 20$ for the first product and $\bar{Y} = 22$ for the second product per week. Assuming that the number of sales per week on the i^{th} product has a Poisson distribution with mean of $\theta_i = 1, 2$. Test the hypothesis that $H_0: \theta_1 = \theta_2$ $H_1: \theta_1 \neq \theta_2$. Take $a = 0.01$ [12 Marks]

(b) A sample of size 1 is taken from an exponential distribution with parameter θ where $X \sim G(1, \theta)$. To test the hypothesis $H_O: \theta = 1$ against $H_1: \theta > 1$, a non-randomised test.

$\emptyset(x) = \begin{cases} 1, if\ x > 2 \\ 0, if\ x \leq 2 \end{cases}$ was used. Find the size a of the test and the power function of this test.

[8 Marks]

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