

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION
(ARTS,SCIENCE)

MATH 420: PARTIAL DIFFERENTIAL EQUATIONS I

STREAMS:

TIME: 2 HOURS

DAY/DATE: MONDAY 4/12/2017

2.30 P.M – 4.30 P.M

INSTRUCTIONS:

- Answer question one and any other two questions

QUESTION ONE

(a) Define a partial differential equation and give an example. [2marks]

(b) Find the integral curves of the equation.

$$\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{xyz - 2x^2} \quad [6marks]$$

(c) Show that the equation is integrable.

$$(x^2z - y^3) dx + x^2y^2 dy + x^3 dz = 0 \quad [4marks]$$

(d) Show that $V = f(y - 3x)$, where f is an arbitrary function is the general solution to the PDE.

$$\frac{dv}{dx} + \frac{3dv}{dy} = 0 \quad [4marks]$$

(e) Find the P.D.E associated with $2z(ax + y)^2 + b$ by eliminating the constants a and b . [5marks]

(f) State the necessary conditions for the equation $P \frac{dz}{dx} + Q \frac{dz}{dy} = R$ to be classified as

- (i) Linear
- (ii) Semi – linear
- (iii) Quasi – linear [3marks]

QUESTION TWO (20MARKS)

(a) Find the partial differential equation arising from $\widehat{\phi}(x+y+z, x^2 + y^2-z) = 0$. [5marks]

(b) Solve the partial differential equation using lagrange’s method. [9marks]

$$\frac{y^2zb}{x} - Xzq = y^2 \quad [9marks]$$

(c) Solve the momogeneous D.E $(yz+z^2) dx + xydz = zdy$. [6marks]

QUESTION THREE (20MARKS)

(a) Find the ortholonial trajectories on the surface $x^2 + y^2 fyz + d = 0$ and its intersection with the planes $z = c$ where c is a parameter. [8marks]

(b) Show that the simultaneous equations

$$p_1dx + Q_1 dy + R_1 dz = 0$$

$$p_2dx + Q_2 dy + R_2 dz = 0$$

Can be reduced to the form

$$\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R} . \text{ Hence solve}$$

$$\frac{dx}{x(z-y)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} \quad [8marks]$$

(c) show that $v = f(y - 3x)$ where f is an arbitrary function is the general solution to the D.E and hence find the particular solution which satisfied the condition $v = (0,y) = 4 \sin y$. [4marks]

QUESTION FOUR (20MARKS)

(a) Find the complete integral solution of the following non linear differential equations.

(i) $P^2 - q^2 = 16$ [7marks]

(ii) $px + qy + p_q = z$ [4marks]

(b) Find the integral curves of the equation using the multipliers (1,1,1). [9marks]

$$\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$$

QUESTION FIVE (20MARKS)

(a) Consider the non linear P.DE $f(x,y,z,p,q) = 0$ where z is a function of x and y . Write down the general auxiliary equation of the char pit method. [2marks]

(i) Solve the differential equation using char pit method.

$$P^2 - xp - q = 0$$

(b) Find the orthogonal trajectories on the surface on the surface $z(x-y) = 1$ of the conics in which it is cut by the system of planes $x-y + z = c$ where c is a parameter.

[8marks]
