

UNIVERSITY EXAMINATIONS

## FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

## MATH 348: OPERATIONS RESEARCH

STREAMS:
DAY/DATE: TUESDAY 05/12/2017

## TIME: 2 HOURS

2.30 P.M. - 4.30 P.M.

INSTRUCTIONS:
(i) Answer question 1 and any other two
(ii) Show all your workings

## QUESTION ONE (30 MARKS)

(a) Define the following terms
(i) Sensitivity analysis [2 marks]
(ii) Degeneracy
[2 marks]
(iii) Optimal solution [2 makers]
(b) Differentiate between balanced and unbalanced transportation problem. [3 marks]
(c) A manager has three jobs to be assigned to three of his clerical staff. Clerical staff differs in efficiency. The efficiency is a measure of time taken by them to do various jobs. The matrix given below shows the time taken by each person to do a particular job.

| Jobs | Men (Time take to do job in hours) |  |  |
| :--- | :--- | :--- | :--- |
|  | $x$ | $y$ | $z$ |
| A | 10 | 27 | 16 |
| B | 14 | 28 | 7 |
| C | 36 | 21 | 16 |

(i) Assign the duty to the staff using Hungarian method. [6 marks]
(ii) Find the minimum total time taken by the staff. [2 marks]

## MATH 348

(d) Consider the linear programming problem below maximize $Z=4 x,+3 x_{2}$ Subject to
$2 x_{1}+3 x_{2} \leq 6$
$-3 x_{1}+2 x_{2} \leq 3$
$3 x_{2} \leq 5$
$2 x_{1}+x_{2} \leq 4$
$x_{1}, x_{2} \geq 0$
(i) Solve graphically [4 marks]
(ii) State the type of feasible region displayed [1 mark]
(iii) Label the redundant constraint [1 mark]
(iv) Explain the meaning of redundancy in management. [2 marks]
(e) (i) Explain briefly limitations of operations research. [3 marks]
(ii) State the primary objective of operations research. [2 marks]

## QUESTION 2 (20 MARKS)

(a) Consider the transportation problem represented in the table below. The transport cost is in (dollars)

Destination

| Origin | A | B | C | D |  |
| :--- | :--- | :--- | :---: | :--- | :--- |
| 1 | 4 | 6 | 8 | 6 | 700 |
| 2 | 3 | 5 | 2 | 5 | 400 |
| 3 | 3 | 9 | 6 | 5 | 600 |
| Demand | 400 | 450 | 350 | 500 | 1700 |

Find the initial basic feasible solution using
(i) The least cost method.
(ii) The Vogel's approximation method
(b) (i) State the duality theorem and explain the importance of duality. [4 marks]
(ii) Write the dual of the linear programming problem below

Maximize $P=3 x+y+5 z$
Subject to
$2 x+4 y+3 z \leq 80$
$x+y+z \leq 40$
$x+y+2 z \leq 40$
$x, y, z \geq 0$

## MATH 348

## QUESTION 3 (20 MARKS)

(a) A company has five jobs $\mathrm{v}, \mathrm{w}, \mathrm{x}, \mathrm{y}$ and z and five machines $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E . The given matrix shows the returns in shillings of assigning a job to a machine. Using Hungarian technique assign the jobs to machines so as to maximize the total returns

|  | Machines |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Jobs | A | B | C | D | E |  |
| V | 5 | 11 | 10 | 12 | 4 |  |
| W | 2 | 4 | 6 | 3 | 5 |  |
| X | 3 | 12 | 5 | 14 | 6 |  |
| Y | 6 | 14 | 4 | 11 | 7 |  |
| Z | 7 | 9 | 8 | 12 | 5 |  |

(b) State four characteristics of a good model. [4 marks]
(c) State and explain the importance of the conditions in simplex method
(i) Optimality condition
[4 marks]
(ii) Feasibility condition
[4 marks]

## QUESTION 4 (20 MARKS)

(a) Use simplex method to

Minimize $W=3 y,+2 y_{2}$
Subject to $\quad y_{1}+3 y_{2} \geq 6$

$$
2 y_{1}+y_{2} \geq 3
$$

$$
\begin{equation*}
y_{1}, y_{2} \geq 0 \tag{6marks}
\end{equation*}
$$

(b) A company has three factors located in three cities $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$. These factors supplies consignments to four dealers, A B C and C. the dealers are spread all over the country. The production capacity of these factories is 1000,700 and 900 units per month respectively. The net return per unit product is given in the following table

| Dealers |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Factory | A | B | C | D | Capacity |
| X | 6 | 6 | 6 | 4 | 1000 |
| Y | 4 | 2 | 4 | 5 | 700 |
| Z | 5 | 6 | 7 | 8 | 900 |
| 9000 | 800 | 500 | 400 | 2600 |  |

## Requirement

Determine a suitable allocation to maximize the total return (use the least cost method to obtain basic feasible solution then MODIto determine the optimum solution)[14 marks]

## MATH 348

## QUESTION FIVE (20 MARKS)

(a) Use the duality method to solve the linear programming problem below

$$
\begin{array}{ll}
\text { Minimize } & C=10 x_{1}+8 x_{2} \\
& x_{1}+2 x_{2} \geq 2 \\
& x_{1}+x_{2} \geq 5 \\
& x_{1} \geq 0 x_{2} \geq 0 \tag{5marks}
\end{array}
$$

(b) Use graphical method to obtain the optimum solution to the linear programming problem below

$$
\begin{array}{ll}
\text { Maximize } & f=2 x_{1}+6 y \\
& 2 x+5 y \leq 30 \\
& x+y \leq 25 \\
& x+y \leq 11 \\
& x \geq 0, y \geq 0
\end{array}
$$

(c) State and explain briefly four advantages of operations research.
(d) A company is producing a single product and is selling it through five agencies situated in different cities. All of a sudden there is a demand for the product in another five cities not having any agencies of the company. The company is faced with the problem of deciding how to assign the existing agencies to dispatch the product. The distance between surplus and deficit cities are given in the following distance matrix.

| Surplus/deficit <br> cities | Programmes |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | I | II | III | IV | V |
| A | 160 | 130 | 175 | 190 | 200 |
| B | 135 | 120 | 130 | 160 | 175 |
| C | 140 | 110 | 155 | 170 | 185 |
| D | 50 | 50 | 80 | 80 | 110 |

Determine the optimum assignment schedule.

