CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 344: THOERY OF ESTIMATION

STREAMS: **TIME: 2 HOURS**

DAY/DATE: TUESDAY 05/12/2017 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS:

QUESTION ONE (30 MARKS)

(a) Let $Z = a^2x + b^2$ and that X is a random variable from a given population, and that a and b are constants

Find

(i) Var (Z). isVar (Z) an unbiased estimator of Var (X)? Explain your answer. [3 marks]

(ii) For Var (Z) to be an unbiased estimator of Var (X), what is the value of a? [1 mark]

(b) Given a random sample of size n from a gamma population, use the method of moments to obtain the formula for estimating $\propto$ and β.

(Note: for a gamma population $E(X) = \mu_1^{'} = \propto \beta$ and $Var(X) = \mu_2^{'} = \propto (\propto +1)\beta^2$). [5 marks]

(c) Studies have shown that a random variable X, the processing time required to do a multiplication in a 3-D computer is normally distributed with mean μ and standard deviation 2 microseconds. A random variable of 16 observations is to be taken. These are the data obtained: 42.65, 45.13, 39.32, 44.44, 41. 63, 41.54, 41.59, 45.68,46.50, 41.35, 44.37, 40.27, 43.87, 43.79, 43.28, 40.7

(i) What is the distribution of the random variable X? [1 mark]

(ii) Based on the data given above, find the unbiased estimate for μ. [3 marks]

(d) From your results obtained in c (ii) above, find a 95% confidence interval for μ. Would you be surprised to read that the average time required to process a multiplication on this system is 42.2 microseconds? Explain based on the confidence interval obtained.

[4 marks]

(e) Consider a random variable X with a density given by

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, for -\infty < x < \infty$

Show that $\bar{x}$ is a minimum variance unbiased estimator forμ. [4 marks]

(f) (i) By use of moments, show that $Var(X) = \mu_2^{'} - \mu_1^{'}$ [3 marks]

(ii) Given that $S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$, show that S^2 is a biased estimator of $Var(X)$.

[4 marks]

(iii) Differentiate between a parameter and a statistic. [2 marks]

QUESTION TWO (20 MARKS)

(a) Consider a random variable X with density given by

$f(x) = (1+\theta)x^{\theta}, 0 < x < 1;\ \theta > -1$

(i) Show that $\int_0^1 f(x)dx = 1$ regardless of the specific value chosen for θ.[3 marks]

(ii) Find $\mu_1^{'}$ [3 marks]

(iii) Find the method of moments estimate for θ based on the data given below:

0.5, 0.3, 0.1, 0.1 and 0.2 [3 marks]

(iv) Find the maximum likelihood estimate for θ based on the data in (a) (iii) above. Does this value agree with the method moments estimate in (a) (iii) above?

[5 marks]

(b) The summary statistics given are results of data analysis on the study of relative humility (%), X, and solvent evaporation (%), $Y: n = 25$,

$\sum x^2 = 76308.53,\ \sum x = 1314.9, \sum y^2 = 2286.07, \sum xy = 11824.44, \sum y = 235.7$

Use the summary statistics given to:

(i) Estimate $\beta_0$and β_1 [4 marks]

(ii) Fit a linear regression model and estimate Y when $X = 29.75$ [3 marks]

QUESTION THREE

(a) Let $\hat{\theta}$ be an estimator of a parameter θ. Show that the mean square error M.S.E $(\hat{\theta})$ of θ is given by

M.S.E $(\hat{\theta}) = E(\hat{\theta}-\theta)^2 = var\,(\hat{\theta}) + [b(\hat{\theta})]^2$ where $b(\hat{\theta})$ is the bias of $\hat{\theta}$. [5 marks]

(b) Let $X_1, X_2, \dots, X_n$ be a random sample from a distribution with pdf $f(x)$ given by

$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x} & 0 < x \\ 0 & elsewhere \end{cases}$$

Find the estimator of λ using the method of moments. [5 marks]

(c) Suppose $X_1, X_{2,} \dots, X_n$ constitutes a random sample from a population with probability density function $f(x|\theta) = \begin{cases} \frac{1}{\theta} & 0 < x < \theta, \quad 0 < \theta \\ 0 & elsewhere \end{cases}$

Where θ is an unknown parameter.Let $T = Max\,(x_1, x_2, \dots, x_n)$. Show that

$\theta = \left(1 + \frac{1}{n}\right) T$ is an unbiased estimator of θ. [10 marks]

QUESTION FOUR

(a) Suppose $x_1, x_2, \dots, x_n$ is a random sample drawn from a population with pdf $f(x|\theta)$ where θ is an unknown parameter. Let $d(\tilde{x}) = d(x_1, x_2 \dots, x_2)$ be any unbiased estimator of a function $g(\theta)$ of the parameter θ. If $f(x|\theta\,)$satisfies certain regulatory conditions, then the Cramer-Rao inequality theorem gives the lower bound of the variance of $d(x)$.

(i) State the regularity conditions. [4 marks]

(ii) State and prove Cramer-Rao inequality theorem. [10 marks]

(b) Consider the normal distribution with mean μ and variance σ^2 that is $\backsim N[\mu, \sigma^2]$ where both μ and σ^2 are unknown. Let S_1^2 and S_2^2 where $S_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ and

$S_2^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ be two estimators of σ^2.Which one of the two is more efficient?

[6 marks]

QUESTION FIVE

(a) Suppose X_1, X_2, X_3 form a random sample of size 3 from a bernoull distribution with parameter θ, show that the statistic T is given by

$T\ = X_1 + 2X_2 + 3X_3$is not sufficient for θ [6 marks]

(b) Let x be a poisson variable with parameter λ.Let$X_1, X_2, \ldots, X_n$ be a random sample on X. find the minimum variance best unbiased estimator of λ if it exists. [10 marks]

(c) Suppose $X \sim N[\mu, \sigma^2]$ with σ^2 known. If $X_1, X_2, \ldots, X_n$ is a random samplefrom X, construct a $(1-\propto)$ 100% confidence interval for μ. [4 marks]
