CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE/ARTS, BACHELORS OF SCIENCE, BACHELORS OF ARTS(MATHS-ECONS), BACHELORS OF SCIENCE(ECON STATS)

MATH 316: LINEAR ALGEBRA II

STREAMS:BED(ARTS,SCI),BSC (ECONSTAT,MATH &EECONS)
TIME: 2HRS

DAY/DATE: MONDAY 11/12/2017 11.30 A.M – 1.30 P.M

INSTRUCTIONS:

- **Answer Question ONE and any other TWO Questions**
- **Sketch maps and diagrams may be used whenever they help to illustrate your answer**
- **Do not write anything on the question paper**
- **This is a closed book exam, No reference materials are allowed in the examination room**
- **There will be No use of mobile phones or any other unauthorized materials**
- **Write your answers legibly and use your time wisely**

QUESTION ONE: (30 MARKS)

a) Consider the following two bases of $\mathbb{R}^2$
$\{e_1 = (1,0), e_2 = (0,1)\}$ and $\{f_1 = (1,1),\ f_2 = (-1,0)\}$ .Find the transition matrix P from the basis $\{e_i\}$ to the basis $\{f_i\}$ and Q from the basis $\{f_i\}$ to the basis $\{e_i\}$ (5 marks)

b) Given that $A = \begin{bmatrix} 2 & -3 & 3 \\ 3 & -4 & 3 \\ 6 & -6 & 5 \end{bmatrix}$, find the eigenvalues of A^3 (5 marks)

c) Find the symmetric matrix that correspond to the following quadratic form
$q(x, y, z) = xy + y^2 - 4xz + z^2$ (3 marks)

d) Let A be an nxn matrix over a field K. show that the mapping $f(X, Y) = X^T AY$ is a bilinear form on K^n (3 marks)

e) Prove that similar matrices have the same characteristic polynomial. (3 marks)

f) State how elementary row operations affect the determinant of a square matrix, hence or otherwise show that if two rows are equal the determinant is zero. (4 marks)

g) Show that if $A = \begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix}$, then find A^5 (3 marks)

h) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$ (4 marks)

QUESTION TWO (20 MARKS)

a) Let f be a bilinear form on R^2 defined by
$f[(x_1, x_2), (y_1, y_2)] = 3x_1y_1 - 2x_1y_2 + 4x_2y_1 - x_2y_2$. Find

i. The matrix A of f in the basis $\{u_1 = (1,0), u_2 = (1,1)\}$

ii. The matrix B of f in the basis $\{v_1 = (2,1), v_2 = (1,-1)\}$

iii. The change of basis matrix P from the basis $\{u_i\}$ to the basis $\{v_i\}$ and verify that $B = P^T AP$.

(12 marks)

b) Let A be the matrix

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 5 & 3 \\ 1 & 3 & 2 \end{bmatrix}$$

Apply diagonalization algorithm to obtain a matrix P such that $D = P^T AP$

(4 marks)

c) Consider the basis U of R^4 spanned by the vectors $\{v_1 = (1,1,1,1), v_2 = (1,1,2,4), v_3 = (1,2,-4,-3)\}$, use the Gram Schmidt formula to find an orthonormal basis. (4 marks)

QUESTION THREE (20 MARKS)

a) Given that $A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 5 & -3 & 1 \\ 1 & 6 & -2 & 4 \\ 0 & 1 & -3 & 7 \end{bmatrix}$ determine the number n_k and the sum S_k of principal minors of order 1, 2 and 4. (7 marks)

b) Let $A = \begin{bmatrix} 2 & -3 & 3 \\ 3 & -4 & 3 \\ 6 & -6 & 5 \end{bmatrix}$

i. Find the characteristic polynomial of A. (3 marks)

ii. Find all the eigenvalues of A and their corresponding eigenvectors. (6 marks)

iii. Is A diagonalizable? If yes, Determine the matrices P and D such that $D = P^{-1}AP$ such that D is diagonal. (1 mark)

iv. Find a matrix B such that $B^2 = A$ (3 marks)

QUESTION FOUR (20 MARKS)

a) State Cayley-Hamilton theorem and verify using a linear operator $T: R^2 \to R^2$ defined by $T(x_1, x_2) = (4x_1 - 3x_2, x_1 + 5x_2)$

(6 marks)

b) Find the characteristic polynomial and hence the minimal polynomial of the matrix

$$A=\begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$ (6 marks)

c) Consider the basis of R^3 consisting of the vectors $y_1 = (1,0,1)$, $y_2 = (2,-1,3)$ and $y_3 = (-1,1,1)$ and a non singular matrix $P = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ -1 & 2 & 4 \end{bmatrix}$.Find the vectors x_1, x_2, x_3 which form a basis for $\mathbb{R}^3$ so that P is the transition matrix from the basis consisting of the vectors x_1, x_2 and x_3 to the basis formed by y_1, y_2 and y_3 (8 marks)

QUESTION FIVE (20 MARKS)

a) Let A=$\begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$ be a 5-square matrix and $I = \{1,2,4\}$ and $J = \{1,3,5\}$ be row and column indices of A respectively.

i. find the minors A(I;J), A(I';J') and their corresponding signed minors (2 marks)

ii. find the minimal polynomial for A (3 marks)

b) i. Define a complex inner product space. (2 marks)

ii. Let V be a complex inner product space, verify that

$< u, av_1 + bv_2 >= \bar{a} < u, v_1 > + \bar{b} < u, v_2 >$ (2 marks)

iii. Suppose $< u, v >= 3 + 2i$, evaluate $< (3-6i)u, (5-2i)v >$ (3 marks)

c) Consider the quadratic form $q(x, y) = 3x^2 + 2xy - y^2$ and the linear substitution $x = s - t$ and $y = s + t$

i. Rewrite $q(x, y)$ in matrix notation and find the matrix notation and find the matrix A representing $q(x, y)$ (1 mark)

ii. Rewrite the linear substitution using matrix notation and find the matrix P corresponding to the substitution (3 marks)

iii. Write the quadratic form $q(s,t)$ (2 marks)

iv. Verify part iii above using direct substitution (2 marks)
