CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EMBU CAMPUS

EXAMINATION FOR THE AWARD OF DIPLOMA IN EDUCATION

MATH 0313: VECTOR ANALYSIS

STREAMS: DIP. EDUC **TIME: 2 HOURS**

DAY/DATE:

INSTRUCTIONS:

- **ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS**
- **ADHERE TO THE INSTRUCTIONS ON THE ANSWER BOOKLET**
- **DO NOT WRITE ON THE QUESTION PAPER**

QUESTION ONE (30 MARKS)

(a) State the divergent theorem. [2 marks]

(b) If $R(t) = x\vec{i} + y\vec{j} + z\vec{k}$

Show that $\frac{d\vec{R}}{dt} = \frac{dx\vec{i}}{dt} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$ [5 marks]

(c) Define an implicit function. [2 marks]

(d) Given that

$F = x^2y + y^2z + xz$ [4 marks]

Find $\frac{dz}{dx}$ and $\frac{dz}{dy}$

(e) Find the Laplace transform of $f(t) = e^{at}$ [3 marks]

(f) Define

Gradient ϕ [2 marks]

(g) (i) Let $\phi(x, y, z) = 3x^2y - y^3z^2$ find $\vec{\nabla}\phi$ at the point $(1, -2, -1)$ [4 marks]

(ii) Let $\vec{A} = x^2z\vec{\imath} - 2y^3z^2\vec{\jmath} + xy^2\overrightarrow{zk}$ find divergence $\vec{A}$ at $(1, -1, 1)$ [5 marks]

(h) Explain what is meant by a Curl of a vector. [3 marks]

QUESTION TWO (20 MARKS)

(a) Let $\vec{\nabla}= \vec{w}x\vec{r}$. Prove that

$\vec{w} = ½Curl\,\vec{r}$ [7 marks]

(b) Define a conservative vector field. [2 marks]

(c) Show that $\vec{F} = (2xy + z^3)i + x^2\vec{y} + 3xz\vec{k}$ is a conservative vector field. [6 marks]

(d) Find the scalar potential. [5 marks]

QUESTION THREE (20 MARKS)

(a) Evaluate the surface integral. [10 marks]

$\iint \vec{F}\tilde{n}\overrightarrow{ds}$ where

$\vec{F} = 4xz\vec{\imath} - y\vec{\jmath} + yz\vec{k}$ and S is the surface of the cube bounded by
$x = 0, x = 1, y = 1, z = 1$

$x = 0,\ x = 1,\ y = 0, y = 1, z = 0, z = 1$

(b) Find $\iiint \vec{F}$ dr where [10 marks]

$\vec{F} = 2xz\vec{c} - x\vec{\jmath} + y^2\vec{k}$

$2xz\vec{\imath}$

Where v is the solid region bounded by the forces
$x = 0,\ y = 0,\ y = 6,\ z = x^2, z = 4,\ \ x = 2$

QUESTION FOUR (20 MARKS)

(a) State the Green's theorem. [2 marks]

(b) Verify Green's theorem in the plane for the function $\oint_c \left(xy+y^2\right)dx + x^2 dy$ where c is a closed curve bounded by $y = x_1 y = x^2$

(c) Show that the polynomial $y = X^2 - 4x = f(x)$ is continuous and differentiable for all x in the interval $-\infty < x < \infty$

QUESTION FIVE (20 MARKS)

(a) Suppose that $f(x) = X^{½} - X^{½}$ on $(\partial, 1)$. find the number C that satisfies the conclusion of Rolle's theorem. [7 marks]

(b) State the stokes theorem. [2 marks]

(c) Verify stokes theorem for $\vec{A} = (2x - y)\vec{c} - yz\vec{j} - y^2 z\vec{k}$ where S is the upper half surface of the plane $x^2 + y^2 + z^2 = 1$ and C its boundary. [11 marks]
