CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, ART & EDUCATION

MATH 241: PROBABILITY AND STATISTICS I

STREAMS: B.Sc B.Ed BA **TIME: 2 HOURS**

DAY/DATE: FRIDAY 8/12/2017 **8.30 A.M - 10.30 A.M.**

INSTRUCTIONS:

- Answer Question ONE and any other TWO Questions.

QUESTION ONE [30 MARKS]

(a) Define the following terms as used in statistics;
 (i) Random experiment
 (ii) Discrete random variable
 (iii) Continuous random variable
 (iv) Sample space [4 Marks]

(b) A discrete random variable X has a probability distribution given by

$$f(x) = \begin{cases} \frac{2}{3}\left(\frac{1}{3}\right)^x & x = 0, 1, 2, 3 \ldots \\ 0 & , otherwise \end{cases}$$

 (i) Show that f(x) is probability distribution function. [2 Marks]

 (ii) Find the cumulative distribution function (cdf) of X. [2 Marks]

 (iii) Determine the m.g.f of X. [2 Marks]

 (iv) Using (iii) compute E(X) and standard deviation of X. [3 Marks]

(c) The loaves of ryre bread distribution to local stores by a certain bakery have an average length of 30 cm and standard deviation is 2cm. Assuming the length are normally distributed, what is the probability of the loaves being;
 (i) Longer than 31.5 cm
 (ii) Between 29.3 and 33.5 cm

(iii)Shorter than 25.5 cm [6 Marks]

(d) A random variable X has a pd.f $f(x) = \begin{cases} k[x^{½} - x], & 0 \leq x \leq 1 \\ 0 & , \text{elsewhere} \end{cases}$

Find,

(i) The value of k [2 Marks]
(ii) The mean and variance of X [3 Marks]
(iii) $E(2x+3)^2$ [3 Marks]

(e) A random variable X has the following probability distribution functions

x	1	2	3	4	5	6	7
f(x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find the value of k [2 Marks]
(ii) Evaluate $p(0 < x < 5)$ [2 Marks]

QUESTION TWO [20 MARKS]

The m.g.f of random variable X is given by $M_x(t) = \left[\frac{7}{10}e^t + \frac{3}{10}\right]^9$

Find

(i) E(X) and Var (x)
(ii) m.g.f of the random variable $Y = 6x + 5$
(iii) $P[X = 3 \; or \; X = 6]$ [10 Marks]

(b) Suppose that 35% of item produced by a factory are defective, if 20 items are inspected, what is the probability that the number of defective items is between 12-15 inclusive. Use normal approximation method. [10 Marks]

QUESTION THREE

(a) The probability density function f(x) of a continuous random variable x is given by

$$f(x) = \begin{cases} 2(3-x) & 2 \leq x < k \\ 0 & , \text{elsewhere} \end{cases}$$

Where k is a constant

(i) Find the value of k for f(x) to be a valid probability density function. [4 Marks]
(ii) Evaluate $p(2.5 \leq x)$ [3 Marks]

(b) A discrete random X has probability mass function f(x) given by

$$f(x) = \begin{cases} \frac{1}{50}x(x+2), & x = 1, 2, 3, 4 \\ 0 & , \text{otherwise} \end{cases}$$

Determine

(i) Mean of X [2 Marks]

(ii) Variance of X [3 Marks]

(iii) $E(2X^2 - 3X + 4$ [2 Marks]

(c) (i) Suppose the probability density function f(x) of a continuous random variable X is given by $f(x) = \begin{cases} \frac{1}{12}(2x-1) & 1 \leq x \leq 4 \\ 0 & , elsewhere \end{cases}$

Find the median of X [3 Marks]

(ii) Find the mode of a continuous random variable X whose probability density function f(x) is given by $f(x) = \begin{cases} 6x(1-x) & 0 < x1 \\ 0 & , elsewhere \end{cases}$

QUESTION FOUR

(a) (i) A random variable X has a poisson distribution such that $2P(X = 1) = 1P(X = 2)$
Find P(x=3) [5 Marks]

(ii) Given the probability distribution function f(x) of a random variable X having a poisson distribution as $f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & x = 0, 1, 2 \ldots \\ 0, & otherwise \end{cases}$

Derive the mean and variance of X. [8 Marks]

(b) The probability that a student will be caught cheating during an examination is 0.9. find the probability that;

(i) A given student will be caught cheating for the first time during his third attempt to cheat. [2 Marks]

(ii) A student will be caught cheating before his third attempt to cheat. [2 Marks]

(c) The moment generating function of a random variable X having a gamma distribution with parameters $\propto$ and β is given by $M_x(t) = \left[\frac{1}{1-\beta t}\right]^{\propto}$

Use $M_x(t)$ to find the mean of X. [4 Marks]

QUESTION FIVE [20 MARKS]

(a) If X has an exponential distribution given by

$$f(x,\lambda) = \begin{cases} \lambda e^{-\lambda x} & 0 < x < \infty \\ 0, & elsewhere \end{cases}$$

Find (i) $M_x(t)$ (ii) E(X) and (iii) var (X) [6 Marks]

(b) A random variable X has a p.d.f

$$f(x) \begin{cases} K\left(\frac{1}{4}\right)_x & \text{x} = 1, 2, 3 \\ 0, & elsewhere \end{cases}$$

Find(i) The value of k

(ii) The f.m.g.f of X

(iii)Hence find the mean and variance X [7 Marks]

(c) Given that $f(x) = ⅓ \quad -1 < x < 2$

$0, \quad elsewhere$

Find the third moment about the mean (μ_3). [4 Marks]

(d) State the characteristics of a Bernoulli distribution. [3 Marks]

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