CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DEGREE OF BACHELOR OF EDUCATION (ARTS, SCIENCE) ,BACHELOR OF ARTS (MATHS ECONS), BACHELOR OF SCIENCE (ECON STATS)

MATH 222: VECTOR ANALYSIS

STREAMS: **TIME: 2 HOURS**

DAY/DATE: MONDAY 11/12/2017 **11.30 A.M – 1.30 P.M**

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Show that addition of two vectors is commutative (2 marks)

(b) Determine the unit vector perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\vec{j} - 3\vec{k}$ and $\vec{B} = 4\hat{i} + 3\vec{j} - \vec{k}$ (3 marks)

(c) If $\vec{v} = \vec{\omega} \times \vec{r}$, prove that $\vec{\omega} = \frac{1}{2}(\nabla \times \vec{v})$, where $\vec{\omega}$ is a constant vector (4 marks)

(d) Using vector method find the area of the triangle having vertices $P(3,-1,2), Q(1,-1,-3), R(4,-3,1)$ (4 marks)

(e) Given $\phi(x,y,z) = 2x^3y^2z^4$, calculate;

(i) $Div(grad\,\phi)$ (3 mks)

(ii) $Curl(grad(\phi))$ (2 mks)

(f)) If $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$, evaluate $\int_c \vec{A}.dr$ from $(0,0,0)$ to $(1,1,1)$ along the straight lines from $(0,0,0)$ to $(1,0,0)$, $then\ to$ $(1,1,0)$, $and\ then\ to$ $(1,1,1)$ (5mks)

(g) Given that $\emptyset = 45x^2y$, evaluate its volume integral such that the volume space is bounded by planes $4x + 2y + z = 8,\ x = 0, y = 0, z = 0.$ (5 marks)

(h) Give the transformation equations from cylindrical to rectangular coordinate system (2 marks)

QUESTION TWO: (20 MARKS)

(a) Find the equation for the tangent to the surface $2xz^2 - 3xy - 4x = 7$ at the point (1,-1,2) (5 marks)

(b) (i) Show that $\nabla \cdot \nabla\emptyset = \nabla^2\emptyset$, where $\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$ denotes the Laplacian operator.(4 marks)

(ii) Hence show that $\nabla^2(\ln r) = \frac{1}{r^2}$ (6 marks)

(c) The position vector of a particle in space is given by the equation $\vec{s} = 2\cos 2t\,\hat{\imath} + 2\sin 2t\,\hat{\jmath} + 3t\hat{k}$

(i) Find the equations of the velocity and acceleration at any time t (2 marks)

(ii) Find the initial speed of the particle (1 mark)

(iii)Find the length along the curve of the trajectory from the point (2,0,0) to (2,0, π) (2 marks)

QUESTION THREE: (20 MARKS)

(a) (i) If $\vec{F}$ is a conservative vector field, show that Curl $\vec{F} = 0$ (3 marks)

(ii)show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational (3marks)

(ii) Hence show that the vector $\vec{A}$ in (a (ii)) above can be expressed in as a gradient of a scalar function$\emptyset$ (6 marks)

(iii)If Find the work done in moving an object in this field from (0,1,-1) to $(\frac{\pi}{2}, -1, 2)$ (2marks)

(b) State without proof the Stoke's theorem. Hence evaluate the line integral $\int_C \vec{A} \cdot d\vec{r}$ where $\vec{A} = xz\hat{\imath} + yz\hat{\jmath} + xy\,\hat{k}$ and c curve of the intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$ above the x-y plane (6marks)

QUESTION FOUR: (20 MARKS)

State and verify the Divergence theorem for the flux given by $\vec{F} = x^2\hat{i} + y^2\vec{j} + z^2\vec{k}$ on the surface S bounded by the sphere $x^2 + y^2 + z^2 = 1$ and the planes $z = -1$ and $z = 1$ (20 marks)

QUESTION FIVE: (20 MARKS)

(a) State without proofthe Frenet-Serret formulas (3 marks)

(b) Given the space curve defined by $x = e^t, y = e^t \sin t, z = e^t \cos t$ .Find

(i) The tangent vector $\vec{T}$ (3 marks)

(ii) The principal normal $\vec{N}$ (3 marks)

(iii) The Binormal $\vec{B}$ (2 marks)

(iv)The curvature κ (2 mark)

(c) Verify the Green's Theorem for $\int_C (xy + y^2)dx + x^2dy,$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ (7 marks)
