CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

MATH 125: DISCRETE MATHEMATICS

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 14/12/2017 **2.30 P.M – 4.30 P.M**

INSTRUCTIONS:

- **Answer Question ONE and any other TWO Questions**
- **Sketch maps and diagrams may be used whenever they help to illustrate your answer**
- **Do not write anything on the question paper**
- **This is a closed book exam, No reference materials are allowed in the examination room**
- **There will be No use of mobile phones or any other unauthorized materials**
- **Write your answers legibly and use your time wisely**

QUESTION ONE (30 MARKS

a) Determine the validity of the following argument

S_1 : Some crazy people are dangerous

S_2 : All fanatics are crazy

Conclusion: some fanatics are not dangerous. (3 marks)

b) Prove the proposition that the sum of n positive even integers is $\frac{n(n+1)}{2}$ (4 marks)

c) Find all the integers such that $2 < 8 - 3n \le 18$ (3 marks)

d) Give an example of a non-trivial relation on the set $A = \{1,2,3\}$ which is

i. Both symmetric and antisymmetric (2 marks)

ii. Neither symmetric nor antisymmetric (2 marks)

e) Solve the linear congruence equation $4x \equiv 6(\text{mod}10)$ (4 marks)

f) Find the product of the polynomials $f(x) = 4x^3 - 2x^2 + 3x - 1$ and $g(x) = 3x^2 - x - 4$ over Z_5 (4 marks)

g) Prove that $(a+b)' = a'*b'$ (5 marks)

h) Given a binary on the set of integers given by $a*b = a+b-ab$. show that * is commutative and associative (3 marks)

QUESTION TWO (20 MARKS)

a) Let $A = \{1,2,3\}$ $B = \{a,b,c,d\}$ and $C = \{x,y,z,w\}$. Suppose R and S are relations from A to B and from B to C respectively defined by $R = \{(1,a),(2,a),(2,c),(2,d),(3,b)\}$ and $S = \{(a,x),(a,z),(c,w),(d,y)\}$.

i. Draw an arrow diagram to represent the relation $R°S$ (2 marks)

ii. Show that the product of the matrix representation of R and S has the same representation as the matrix $R°S$ (4 marks)

iii. Find the domain and range of $R°S$ (2 marks)

b) Let $S = \{1,2,\ldots,9\}$ and R be a relation on S defined by $(a,b) \approx (c,d)$ if and only if $a+d = b+c$.

i. Show that $\approx$ is an equivalence relation (6 marks)

ii. Find the equivalence class of [2,5] (2 marks)

c) Use Venn diagrams to determine the validity of the following arguments

S_1: Some innocent people go to Jail

S_2: Mary is innocent

S_3: All people in jail are bad people

Conclusion: Mary is not a bad person. (4 marks)

QUESTION THREE(20 MARKS)

a) Use mathematical induction to prove that 57 divide n =0 $7^{n+2}+8^{2n+1}$ (6 marks)

b) Let a=195 and b=968. Use the division algorithm to find the gcd(a,b) and therefore find integers m and n such that d=am +bn (6 marks)

c) Consider the third order homogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$

i. Find the general solution (4 marks)

ii. Find the initial solution given $a_0 = 3, a_1 = 4, a_2 = 12$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Solve the congruence equation $f(x) = 26x^4 - 31x^3 + 46x^2 - 76x + 57 \equiv 0 \pmod 8$ (10 marks)

b) Consider the Boolean algebra D_{210}

i. Find its elements and draw its lattice diagram (2 marks)

ii. Find the set A of atoms (2 marks)

iii. Find two sub-algebras with 8 elements (2 marks)

iv. Is $X = \{1,2,6,210\}$ a sub-lattice or a sub-algebra? Explain. (2 marks)

v. Do as part (iv) above for $Y = \{1,2,3,6\}$ (2 marks)

QUESTION FIVE (20 MARKS)

a) Prerequisites in a college are partial ordering of available classes. Denote A is a requisite of B by $A \prec B$. Let C be the ordered set of mathematics courses and their prerequisite as shown below.

class	Math 101	Math 201	Math 250	Math 251	Math 340	Math 341	Math 450	Math 500
Prerequisite	None	Math 101	Math 101	Math 250	Math 201	Math 340	Math 201,250	Math 250,251

i. Draw with explanations the Hasse diagram for the partial ordering of the classes (5 marks)

ii. Find all the minimal and maximal elements of C and verify them (4 marks)

iii. Does C have a first or last element? Explain (4 marks)

b) Let E be the equation $x + y + z = 18$. Find the number of non-negative solutions of E such that $x > 7, y > 8, z > 9$. (7 marks)