CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE (PHYSICS)

PHYS 811: MATHEMATICAL PHYSICS

STREAMS: MSC. PSYC **TIME: 3 HOURS**

DAY/DATE: MONDAY 09/12/2019 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (25 MARKS)

a) Find out the differentiability of the function $f(z) = z^*$ [4 marks]

b) Show that the following four matrices form a group under matrix multiplication [5 marks]

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

c) Use Cauchy-Riemann conditions to show that $f(z) = z^2$ is analytic in the entire z-plane [4 marks]

d) Evaluate the integral $f(x) = \int_0^\infty \frac{sinxt}{t} dt$ using the Laplace transform. [6 marks]

e) Find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$ for $y = e^{-x^2}$ at the point x=0.05 from the data of the table given below [6 marks]

x	$y = e^{-x^2}$	Δ	Δ^2	Δ^3	Δ^4
0	1.00000				
0.05	0.99750	-250			
0.10	0.99005	-745	-495		
0.15	0.97775	-1230	-485	+10	
0.20	0.96079	-1696	-466	+19	+9
0.25	0.93941	-2138	-442	+24	+5
0.30	0.91393	-2548	-410	+32	+8

QUESTION TWO (15 MARKS)

a) State and prove the residue theorem [5 marks]

b) Evaluate the integral $\int_0^{2\pi} \frac{\cos 3\theta}{5-4cos\theta} d\theta$ using the residual theorem [10 marks]

QUESTION THREE (15 MARKS)

a) Construct the Green's function for the problem stated mathematically as $\frac{d^2y}{dx^2} - \omega^2 y = f(x)$ where f(x) is a known function and y satisfies the boundary conditions $y(0) = 0$ and $y(L) = 0$ [7 marks]

b) Define the shifting property of the Laplace transform and use it to find the Laplace transform of $e^{-x}\cos x$ [4 marks]

c) Prove the following recurrence relation for Bessel function $J_n'(x) = -\frac{n}{x}J_n(x) + J_{n-1}(x)$

Where the prime denotes the differentiation with respect to x [4 marks]

Given: $J_n(x) = \sum_{r=0}^{\infty}(-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{r!\sqrt{(n+r+1)}}$

QUESTION FOUR (15 MARKS)

a) Solve wave equation [7 marks]

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$

Using the Fourier transform with the initial conditions given as $u(x,0) = f(x)$, and

$$\frac{\partial u(x,0)}{\partial t} = 0$$

b) Define isomorphism and show that the group $(i, -1, -i, 1)$ is isomorphic to the cyclic group $(A, A^2, A^3, A^4 = E)$ [3 marks]

c) Using the table given below, evaluate the integral

$$\int_0^{1.0} \frac{x^3}{e^x - 1} dx$$

By using Simpson's one- third rule [6 marks]

x	$f(x) = \frac{x^3}{e^x - 1} dx$
0	0
0.25	0.055013
0.50	0.192687
0.75	0.377686
1.00	0.581977

QUESTION FIVE (15 MARKS)

a) A sphere of radius a is centred at O. It is cut into two equal halves by the x-y plane. The upper part is maintained at potential $+V_o$ and the lower part at potential $-V_o$. Calculate the potential at a point inside the sphere in the following steps:

i) Write the Laplace's equation satisfied by the potential in spherical polar coordinates and make use of the method of separation of variables to separate it into the $\varphi-, \theta-, and\ r-$ equations. [4 marks]

ii) Solve the $\varphi-, \theta-, and\ r-$ equations. [5 marks]

iii) Make use of the boundary conditions to find the potential. [6 marks]
