CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTERS OF SCIENCE IN ECONOMICS

MSEC 831: MATHEMATICAL METHODS FOR ECONOMISTS

STREAMS: MSC (ECON) **TIME: 3 HOURS**

DAY/DATE: TUESDAY 03/12/2019 **2.30 PM – 5.30 PM**

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER THREE QUESTIONS

QUESTION ONE

(a) A discriminating monopolist producing a single product faced with the following two demand functions from each of the two markets

$P_1 = 25 - 2Q_1$
$P_2 = 40 - {}^{3}/_{2}\, Q_2$

The monopolist has the following total cost function
$C = 60 + 40$
$Q = Q_1 + Q_2$

(i) Find the profit maximizing levels of outputs and prices. [6 marks]

(ii) In the absence of price discrimination, what would be profit maximizing levels of output and prices? [4 marks]

(b) The Leontief inverse for a three-sector economy and the final demand are given below.

$$[1 - A]^{-1} = \begin{bmatrix} 2.4 & 0.6 & 0.3 \\ 2.0 & 3.5 & 2.0 \\ 2.5 & 4.0 & 4.5 \end{bmatrix}$$

$$D = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 50 \end{bmatrix}$$

Compute the sectorial total outputs that will enable the economy to realize the planned final demand. [6 marks]

(c) Find the derivatives of the following functions

(i) $y = \frac{4x^2+3x}{(1+x^2-3x^4+2x)(x^2-4x^3)}$ [3 marks]

(ii) $y = 3^{5x^2-2x^3}$ [2 marks]

QUESTION TWO

(a) Given $Q = 100K^{0.5}\,L^{0.5}$ W = ksh.30 and r = ksh. 40. Find the quantity of labour and capital that the firm should use inoder to minimize the cost of producing 1444 units of output. What's the minimum cost? [6 marks]

(b) Solve the following systems of equation by matrix inverse and Crammer's rule. [6 marks]

$$x_1 + 3x_2 + 3x_3 = 4$$
$$x_1 + 4x_2 + 3x_3 = 6$$
$$x_1 + 3x_2 + 4x_3 = 2$$

(c) You are given the following quadratic form where [6 marks]

$$Q = f(x,y) = -5x^2 + 6xy - 2y^2$$

(i) Present the quadratic form in matrix form

(ii) Find the 2nd order partial derivatives of Q and present them in matrix format form with the elements of matrix presented in an ordered manner.

(d) Find the derivative of $y = a^x$ [2 marks]

QUESTION THREE

(a) Find the stationary point for the following function and determine whether it present a maximum, minimum or saddle point.
$X = f(x,y) = 3x^2 + xy - 2y^2 - 4x - 9y + 10$ [6 marks]

(b) For each of the following functions, find the corresponding Hessian matrices and Hessian determinant. [6 marks]
$Z = f(x,y) = 3x^2 + 4xy - 7y^2$
$Z = f(x_1,x_2) = 18 - 3x_1x_2 - 2x_1^2 + 12X_2^2$

(c) Consider the following quadratic form
$Q = 4X_1^2 + 6x_1x_2 - 3X_2^2$
$Q = 2X_1^2 + 8x_1x_2 + 5X_2^2$

(i) Present each quadratic form Q in matrix format. [4 marks]

(ii) Find the derivating of Q with respect to X_1 *and* X_2 and present answer in matrix form. [4 marks]

QUESTION FOUR

(a) A firm wishing to maximize its output subject to budget has the following production function and cost function

$Q = 40K^{0.25}\ K^{0.75}$
Cost constraint = $4K + 81 = 40$

(i) Set up constrained maximization problem. [1 mark]

(ii) Construct the corresponding Langragian function [1 mark]

(iii) Find critical values of L K and λ [6 marks]

(iv) Confirm whether the critical values present a maximum. [6 marks]

(b) Find the Jacobian matrices and Jacobian determinant of the following systems of equation [6 marks]

$y_1 + 5x_1 + 3x_2$
$y_2 + 8x_1 + 7x_2$

QUESTION FIVE

(a) Given the utility function $\cup = AX^a y^b$ subject to budget constraint $P_x X + P_y Y = B$. Prove that at the point of constrained utility maximization the ratio of prices P_x/P_y must equal to ratio of marginal utilities MU_x/MU_y [10 marks]

(b) Determine whether the following system of two functions are dependent or independent. [4 marks]

$Z_1 = f'(x, y) = 4x^2 - 2xy + 6y^2$
$Z_2 = f^2(x, y) = 6x^2 - 3xy + 9y^2$

(c) The national income model for an open economy is given as;
$\mathrm{Y = C + I + G + X - M}$
$\mathrm{C = a + by^d}$
$\mathrm{I = I + I_0 + I_1Y}$

$$\mathrm{M} = \mathrm{M}_0 + \mathrm{M}_1\mathrm{Y}$$
$$\mathrm{G} = G_0, \mathrm{X} = X_0$$

(i) Present this model in matrix format [2 marks]

(ii) Use matrix inverse to solve for equal national income $(\bar{Y})$, equal consumption $(\bar{C})$ and import $(\bar{M})$, [4 marks]
