FIRST YEAR EXAMINATIONS FOR BACHELOR OF SCIENCE, COMPUTER SCIENCE & APPLIED COMPUTER SCIENCE

COSC 102: DISCRETE STRUCTURES

STREAMS: BSC (COMP SCI & APPLIED COMP. SCI) Y1S2 TIME 2 HOURS


INSTRUCTIONS

- Answer QUESTION 1 and any other TWO QUESTIONS from section B.
- This is a CLOSED BOOK EXAM. No reference materials allowed in examination room. Mobile phones must be switched off.
- Do not write on this question paper
- Write your answers legibly and use your time wisely.
- Scientific, non-programable Calculators may be used.

SECTION A: COMPULSORY

QUESTION 1[30MKS]

a) What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b\}$? [4 marks]
b) Determine the members of the set $S = \{x \mid x$ is the square of an integer and $x < 100\}$ [4 marks]
c) Let be a proposition be, $P :$ I am in Student. , $Q: \text{I love football}$. What is will be: $q \implies p$ (q implies p)? [2 marks]
d) Suppose there are 50 people in a room, how many of them must have their birthday in the same month? [4 marks]
e) Construct the Truth table of the following compound proposition

$$(P \lor \neg Q) \implies (P \land Q)$$

[6 marks]
f) Given that variable names in a programming language can be either a single uppercase letter or an uppercase letter followed by a digit, find the number of possible variable names [4 marks]
g) How many bit strings of length 8 either start with a 1 or end with two bits 00? [2 marks]
h) Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists. Find the number of students:

(i) Only on list A, (ii) only on list B, (iii) on list A or B (or both), (iv) on exactly one list. [4 marks]

SECTION B: ATTEMPT ONLY TWO QUESTIONS FROM THIS SECTION

Question 2 [20mks]

With the use of direct proof or otherwise, prove the following:

(a) The square of an even natural number is even [6 marks]

(b) The square of an odd natural number is odd [4 marks]

(c) The claim that if n is a positive integer, then the quantity \( n^2 + 3n + 2 \) is even [4 marks]

(d) With the use of relevant examples, discuss proof by induction [6 marks]

Question 3 [20mks]

(a) Find the number of permutations of six objects, \{A, B, C, D, E, F\} taking three at a time [8 marks]

(b) A farmer buys 3 cows, 2 pigs and 4 hens from a man who has 6 cows, 5 pigs, and 8 hens. Find the number of choices the farmer has to make [12 marks]

Question 4 [20mks]

(a) Let M, P and C be the sets of students taking Mathematics, Physics and Computer courses respectively in Chuka University. Take \(|M| = 300\), \(|P| = 350\), \(|C| = 450\), \(|M \cap P| = 100\), \(|M \cap C| = 150\), and \(|P \cap C| = 75\), \(|M \cap N \cap P \cap C| = 10\). Determine the number of students taking exactly one of the above courses. [12 marks]

(b) Migingo highland has two kinds of inhabitants, knights and knaves. Knights always tell the truth, and only the truth; Knaves always tell lies, and only lies. John encountered two people on his visit to the highland, A and B. Determine what is A and B if A tells John “B is a Knight” and B “says The two of us are of opposite type” [8 marks]

Question 5 [20mks]

(a) Find the number \( M \) of seven letter words that can be formed using the word “BENZENE”. [8 marks]

(b) Use Binomial theorem to determine the coefficient of \( x^{12} y^{13} \) in the expansion of \( (x+y)^{25} \) [4 marks]

(c) Determine the expansion of \( (x+y)^{4} \) using Binomial theorem [8 marks]