CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS

MATH 923: FUNCTIONAL DIFFERENTIAL EQUATIONS

STREAMS: PhD (APPLIED MATHS) **TIME: 3 HOURS**

DAY/DATE: WEDNESDAY 14/08/2019 **8.30 A.M - 11.30 P.M.**

INSTRUCTIONS:

- **Answer ALL Questions**.

QUESTION ONE

(a) Solve the delay differential equation $\frac{dx}{dt} = -x(t-1)$ given the constant initial data $X(t) = 1$, for $t \in [-1, 0]$ in the interval

(i) $[0, 1]$ [2 Marks]

(ii) $[1, 2]$ [2 Marks]

(iii) $[2, 3]$ [2 Marks]

(b) Analyze the asymptotic behavior of the Neutral differential equation using the spectral approach. $\begin{cases} x^{11}(t) + x^{11}(t-1) = x(t) + x(t-1) \\ X(t) = Q(t), -1 \le t \le 0 \end{cases}$ [9 Marks]

QUESTION TWO

(a) Show that the differential equation $\frac{dy}{dt} = -½y^{-1}$, does not have a solution satisfying $y(0) = 0$, for $t > 0$. [3 Marks]

(b) Show that the differential equation $x^1 = x^{2/3}$, has infinitely many solutions satisfying $x(0) = 0$, on every interval $[0, b]$. [4 Marks]

(c) Determine whether the function $X(x, t) = \frac{x^2+1}{x}.t$ satisfying a Lipchitz condition in the domains

(i) $R_1 = [1, 2]x[0, 1]$ [1 Mark]

(ii) $R_2 = [1, 2]x[0, +\infty]$ [2 Marks]

(iii) $R_3 = [1, +\infty]x[0, T]$ [2 Marks]
(iv) $R_4 = (0, 1)x[0, 1]$ [2 Marks]
(v) $R_5 = (1, 2)x[0, 1]$ [1 Mark]

QUESTION THREE

(a) Consider the system $\frac{du}{dt} = -y(t)u + a(t)/u(t)/p + \frac{1}{(1+t)}q, \mu^{(0)=0}$

Where $y(t) = \frac{c}{1+t^b}, a(t) = \frac{1}{(1+t)\quad m},$

P, q, b ,c and m are positive constants. Give the sufficient condition for the system to converge to zero as $t \to \infty$ [8 Marks]

(b) Consider the system

$$\frac{dx}{dt} = \propto x + 2y$$

$$\frac{dy}{dt} = -2x$$

Where $\propto$ is a constant. Find the critical values of $\propto$ where the change of the qualitative nature of the phase portrait for the system occurs and classify the autonomous system.

[7 Marks]

QUESTION FOUR

(a) Find the maximal solutions for the following initial value problems

(i) $\begin{cases} \frac{dx}{dt} = x \\ x(0) =< \end{cases}$ [5 Marks]

(ii) $\begin{cases} \frac{dx}{dt} = \frac{1}{t^2}\cos\frac{1}{t} \\ X(to) = C, to \neq 0 \end{cases}$ [5 Marks]

(b) Verify whether the tent map $f_2(x) = \begin{cases} 2x, x \in [0, ½] \\ 2(1-x), x \in (½, 1) \end{cases}$

extended to a map

$g: [-1, 1] \to [-1, 1]$ defined by $g(x) = \begin{cases} -f_2(x), x \in [-1, 0] \\ -f_2(x), x \in [0, 1] \end{cases}$

Where the function is a le-limit set. [5 Marks]

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