CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

MATH 841: DESIGN AND ANALYSIS OF EXPERIMENTS

STREAMS: MSC (APP STAT) **TIME: 3 HOURS**

DAY/DATE: TUESDAY 17/12/2019 **8.30 AM – 11.30 AM**

INSTRUCTIONS:

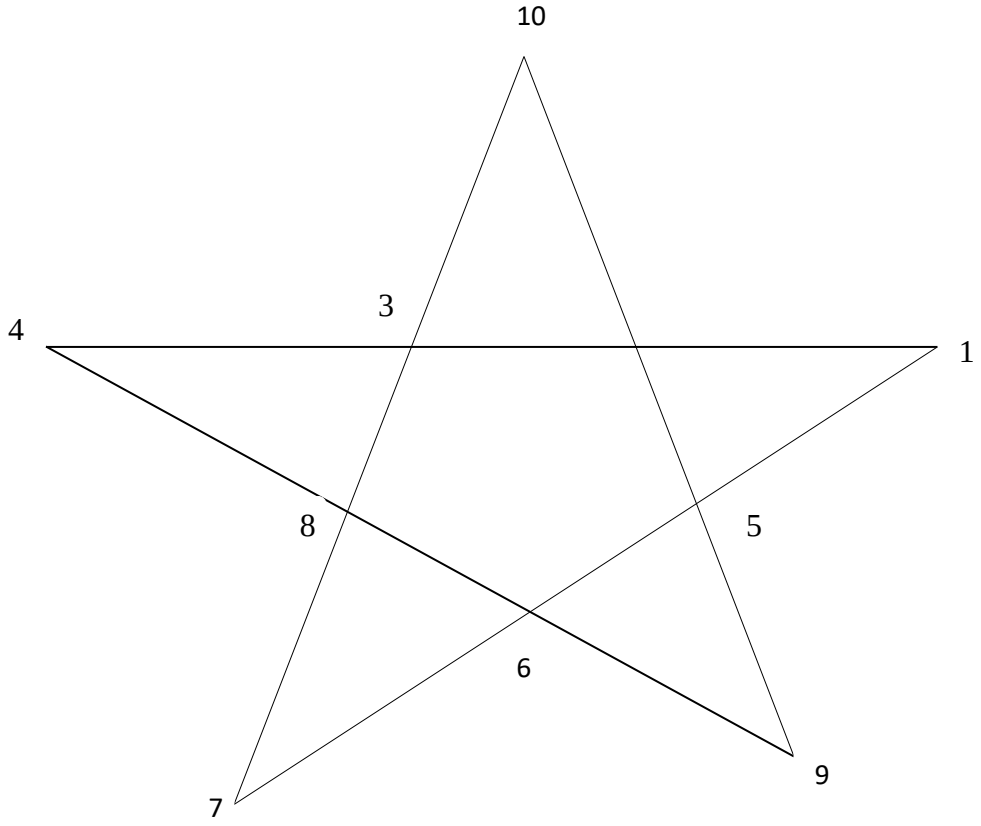
- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (20 MARKS)

a) Explain what is meant by a partially balanced incomplete block design (P.B.I.B) (3mrks)

b) Define the association scheme of P.B.I.B. design with m associate classes. (3mrks)

c) Consider the figure below. Assign a block to elements in a line. Show that the resulting design is P.B.I.B. design with two associate classes. Give primary and secondary parameters of the design and a table of association scheme. (9mrks)



c) Considering the parameters in (b) above, we define $A^{-1} = ((a_{ii})^{-1})$ where $a_{ii} = r\left(1 - \frac{1}{k}\right)\delta_{ii} + \frac{\lambda_i n_i}{k} - \frac{\lambda_1}{k}P_{i1}^{i'} - \frac{\lambda_2}{k}P_{i2}^{i'}$

$\delta_{ii}$is the kronecker's delta and all other symbols retain their usual meaning. Obtain the matrix A^{-1} and state its usefulness. (5mrks)

QUESTION TWO: (20 MARKS)

a) Define the following terms used in factorial experiments

i) Simple effect

ii) Main effect

iii) Interaction between two factors

(3mrks)

b) An experimenter uses a $\frac{1}{4}$ fraction of a 2^5 fractional factorial design to perform an experiment involving factors $A, B, C, D \text{ and } E$. The defining contrast for this design is $I = ABCD = ACE = BDE$

i) List all the aliased groups in this design

ii) What must we assume if we have to estimate all the main effects orthogonally

(5mrks)

c) The following are results of a 2^3 fractional factorial experiment run in a randomized complete block design having blocks of size 4. The interaction ABC is confounded in each of the three replicates. The block totals are in parentheses.

Block	**Replicate**			
	1	2	3	
(1)	7	19	13	39
Ab	39	36	35	110
Ac	31	36	31	98
Bc	27	31	26	84
	(104)	(122)	(105)	331
A	30	33	28	91
B	24	30	19	73
C	21	30	24	75
Abc	39	41	35	115
	(114)	(134)	(106)	354

Obtain

i) All possible effect totals and their S.S.

ii) The ANOVA table

iii) The S.S due to the effect ABC and S.S. due to error ABC

(12 marks)

QUESTION THREE: (20 MARKS)

a) Explain the cases where a split plot design is desired. (3mrks)

b) Suppose we have 4 blocks, 4 levels of factor A and 2 levels of factor B. The data are summarized as below.

Block 1					
	a_1	a_2	a_3	a_4	Total
b_1	3.36	3.11	3.30	2.84	12.64
b_2	2.80	2.37	2.35	2.50	10.02
Total	6.16	5.48	5.65	5.34	22.63

Block 2					
	a_1	a_2	a_3	a_4	Total
b_1	3.71	3.45	2.95	2.99	13.10
b_2	2.55	2.62	2.68	2.53	10.38
Total	6.26	6.07	5.63	5.52	23.48

Block 3					
	a_1	a_2	a_3	a_4	Total
b_1	3.46	3.27	3.07	3.23	13.03
b_2	2.94	2.58	2.33	2.64	10.49
Total	6.40	5.85	5.40	5.87	23.52

Block 4					
	a_1	a_2	a_3	a_4	Total
b_1	3.54	3.07	3.07	2.81	12.49
b_2	2.73	2.68	2.14	2.46	10.01
Total	6.27	5.75	5.21	5.27	22.50

i) Write down the model of analysis

ii) Obtain the ANOVA table for the main plot treatment A

(6mrks)

c) State the necessary and sufficient condition for an incomplete block design to be connected and henceshow that a B.B.I.B design is connected. (5mrks)

d) Consider a B.I.B design with parameters $v = b = 7, r = k = 3$ and $\lambda = 1$. The field plan together with randomization and observations (yields) is as given below;

The letters A, B, C, D, E, F, G represents treatments

Block	Treatments		
1	C(5.8)	F(5.5)	D(7.3)
2	B(10.2)	G(8.8)	F(10.4)
3	A(15.0)	G(15.7)	C(10.1)
4	B(5.8)	D(6.3)	A(8.7)
5	E(11.9)	C(9.3)	B(12.4)
6	E(11.2)	F(10.5)	A(13.7)
7	G(13.1)	E(12.9)	D(7.4)

The unadjusted block totals and unadjusted treatment totals are given as;

$\beta_1 = 18.6, \quad \beta_2 = 29.4, \beta_3 = 40.8, \quad \beta_4 = 20.8, \beta_5 = 33.6, \beta_6 = 35.4, \quad \beta_7 = 33.7,$

$\tau_A = 37.4, \tau_B = 28.4, \tau_C = 25.2, \tau_D = 21.0, \tau_E = 36.0, \tau_F = 26.4$ and $\tau_G = 37.6$

Test the hypothesis H_0: $t_A = t_B = \cdots = t_G$ at 5% level of significance. (5mrks)

QUESTION FOUR: (20 MARKS)

a) Define what is meant by response surface methodology and why is it important in the design and analysis of experiments? (2mrks)

b) The yield from a chemical process is found to be affected by two factors: reaction temperature and reaction time. The current reaction temperature is 230 F and the reaction time is 65 minutes. The experimenter wants to determine the settings of the two factors such that maximum yield can be obtained from the process. The first order model was found to be inadequate for the region near the optimum. He thus o augment the experiment with axial runs to be able to complete a central composite design and fit a second order model to the response. The analysis of the model is summarized below.

Regression Table

Regression Information						
Term	Coefficient	Standard Error	Low Confidence	High Confidence	T Value	P Value
Intercept	94.91	0.168826	94.590146	95.229854	562.176358	0
A:Temperature	0.735248	0.133469	0.482381	0.988114	5.508764	0.000898
B:Time	1.529962	0.133469	1.277095	1.782829	11.463079	0.000009
A • B	0.45	0.188753	0.092392	0.807608	2.384065	0.048591
A • A	-1.520625	0.143129	-1.791795	-1.249455	-10.62414	0.000014
B • B	-2.083125	0.143129	-2.354295	-1.811955	-14.554155	0.000002

ANOVA Table

ANOVA Table					
Source of Variation	Degrees of Freedom	Sum of Squares [Partial]	Mean Squares [Partial]	F Ratio	P Value
Model	5	65.086653	13.017331	91.342551	0.000003
A:Temperature	1	4.324712	4.324712	30.346484	0.000898
B:Time	1	18.726273	18.726273	131.402173	0.000009
A • B	1	0.81	0.81	5.683766	0.048591
A • A	1	16.085568	16.085568	112.87236	0.000014
B • B	1	30.187198	30.187198	211.823438	0.000002
Residual	7	0.997578	0.142511		
Lack of Fit	3	0.520578	0.173526	1.455144	0.352651
Pure Error	4	0.477	0.11925		
Total	12	66.084231			

S =	0.377506	PRESS =	4.4472
R-sq =	98.49 %	R-sq(pred) =	93.27 %
R-sq(adj) =	97.41 %		

i) State the fitted model and discuss the parameter estimates. (4mrks)

ii) Discuss the adequacy of the fitted model. (3mrks)

iii) Obtain the stationary points and classify them. (4mrks)

iv) Predict the value of the maximum response. (2mrks)

(c) Outline the steps of analyzing the lattice design results. [5 marks]
