CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 832: METHODS OF APPLIED MATHS II

STREAMS: MSC **TIME: 3 HOURS**

DAY/DATE: MONDAY 09/12/2019 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS:

- **Answer any FOUR questions**
- **All workings must be shown clearly**
- **Adhere to the instructions on the answer booklet**

QUESTION ONE

(a) Prove the following recurrence relations of Bessel's functions

(i) $J_n'(x) + \frac{n}{x} J_n(x) = J_{n-1}(x)$ [2 marks]

(ii) $J_n'(x) - \frac{n}{x} J_n(x) = -J_{n+1}(x)$ [2 marks]

(b) Show that

$$J_4'(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_o^{(x)}$$

(c) Express $4x^3 + 6x^2 + 7x + 2$, in terms of Legendre polynomials [5 marks]

(d) Find the general solution of the Bessel's equation

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$$ [3 marks]

QUESTION TWO

(a) Write the general function of the associated Legendre equation and find the regular singular points

Hence solve

$(1-x^2)y'' - 2xy' + \left(12 - \frac{4}{1-x^2}\right)y = 0$ about the point $x = 1$ [5 marks]

(b) Obtain the recurrence relation for the hypergeometric function

$x(1-x)\, y'' + [C - (a+b+1)x]y' - abt = 0$ about the point $x = 0$ [4 marks]

(c) Show that the n^{th} spherical Bessel function is given by

$f_n(x) = (-1)^n \; x^n \left(\frac{1}{x}\frac{d}{dx}\right)^n f_o(x)$

Where $f_n(x)$ denotes either

$J_n(x)$ or $N_n(x)$

Hence evaluate $J_2(x)$ and $N_2(x)$ [6 marks]

QUESTION THREE

(a) Solve the integral equation

$y(x) = x + \lambda \int_0^1 (xz + x^2z^2)\, y(z)dt$ [5 marks]

(b) Find the eigenvalues and corresponding Eigen functions of the homogeneous Fredholm equation

$y(x) = \lambda \int_0^{\pi} \sin(x+z)\; y\,(z)dz$ [5 marks]

(c) Solve the following integral equations and discuss the uniqueness of the solutions

(i) $F + \lambda ky = 0$ [1 mark]

(ii) $y = f + \lambda ky$ [1 mark]

(d) Test for an extremum the functional

$I[y\,(x)] = \int_0^1 (xy + y^2 - 2y^2)dx \quad y(0) = 1,\; y(1) = 2$ [3 marks]

QUESTION FOUR

(a) Find the Fourier transform of the function

$g(x) = \begin{cases} 1, & /x/ \leq a \\ 0, & /x/ > a \end{cases}$

Hence find an explicit expression for the solution of the integral equation

$y(x) = f(x) + x\int_{-\infty}^{\infty} \frac{\sin(x-z)}{x-z} y\,(z)\,dz$ [5 marks]

(b) Solve the integral equation

$y(x) = x + \lambda \int_0^1 xz^2 y(z) dz$ by the Neumann series method. Assume $y(x) \;\approx y_0(x) = x$

[5 marks]

(c) Use the Fredholm theory to solve the integral equation

$y(x) = x + \lambda \int_0^1 xy\,(z) dz$ [5 marks]

QUESTION FIVE

(a) Find the shape of the curve of the given perimeter enclosing maximum area [7 marks]

(b) Obtain the solid of maximum volume formed by the revolution of a given surface

[8 marks]
