

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

MATH 823: PARTIAL DIFFERENTIAL EQUATIONS I

STREAMS: M.Sc (APPLIED MATHS)

TIME: 3 HOURS

DAY/DATE: THURSDAY 8/08/2019

8.30 A.M - 11.30 A.M.

INSTRUCTIONS

- Answer any **FOUR** Questions
- Do not write anything on the question paper

QUESTION ONE

(a) Given the pde  $A(xy)U_{xx} + 2B(x_1y)U_{xy} + C(x_1y)U_{yy} = 0$ , show that the characteristics differential equation is given by  $C - 2B \frac{dy}{dx} + A \left(\frac{dy}{dx}\right)^2 = 0$  [6 Marks]

(b) Consider the pde  $U_{xx} + 4U_{xy} + 5U_{yy} = 0$

- (i) Classify the pde [1 Mark]
- (ii) Find the characteristics [3 Marks]
- (iii) Solve pde [5 Marks]

QUESTION TWO

Solve the following pde's by the operator method.

- (i)  $U_{xxx} - 2U_{xxy} = 2e^{2x} + 3x^2y$  [8 Marks]
- (ii)  $U_{xx} + U_{xy} - 6U_{yy} = y \cos x$  [7 Marks]

QUESTION THREE

(a) Given the pde  $U_{xx} = U_t, U(x, 0) = x^2(25 - x^2)$ . Solve the pde by the method of separation of variables. [6 Marks]

(b) A string is stretched and fastened to two point  $l$  apart. Motion is started by displacing the spring from which it is released at a time  $t = 0$ . Find the displacement at any point at a

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distance  $x$  from one end at time  $t$ , given that the equation of vibration of the string is  $U_{tt} = c^2 U_{xx}$  under the following conditions; [9 Marks]

- (i)  $U(0, t) = 0$
- (ii)  $U(0, x) = 0$
- (iii)  $U(l, t) = 0,$
- (iv)  $U(x, 0) = 10 \sin \frac{\pi x}{\tau}$

**QUESTION FOUR**

(a) Express the function  $f(x) = \begin{cases} 1, & \text{when } |x| \leq 1 \\ 0, & \text{when } |x| > 1 \end{cases}$  as a Fourier Integral, hence evaluate  $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda$  [7 Marks]

(b) Solve the differential equation  $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$  given that  $x(0) = 1,$  and  $y(0) = 0$  by Laplace transforms. [8 Marks]

**QUESTION FIVE**

(a) Given the pde  $(1 + y)U_{xx} + 2(1 - x)U_{xy} + (1 + y)U_{yy} = U.$  Determine the values of  $x$  and  $y$  for which the equation is;  
(i) Hyperbolic [2 Marks]  
(ii) Parabolic [2 Marks]  
(iii) Elliptic [2 Marks]

(b) Solve the pde  $U_{xx} + U_{xy} - 6U_{yy} = \cos(2x + y)$  by the D. Operator. [6 Marks]

(c) Evaluate  $\int_0^\infty t e^{-3t} \sin t dt$  by Laplace transforms. [3 Marks]

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