

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE
IN MATHEMATICS

MATH 822: ORDINARY DIFFERENTIAL EQUATION II

STREAMS: MSC (MATH)

TIME: 3 HOURS

DAY/DATE: WEDNESDAY 11/12/2019

2.30 P.M. – 5.30 P.M.

INSTRUCTIONS: Answer any THREE questions

QUESTION ONE (20 MARKS)

- (a) (i) State the condition of exactness for a general 3rd order nonlinear differential equation [10 marks]
- (ii) Test the differential equation for exactness and then solve it
- (b) (i) Convert the second order differential equation $-y'' + x^4y = \lambda y$ into a sturm liouville equation [3 marks]
- (ii) Show that the boundary value problem is a sturm liouville problem [7 marks]
- $$\frac{d^2y}{dx^2} + \lambda y = 0$$
- $$y(1) = 1, \quad y(\pi) = 0$$

QUESTION TWO (20 MARKS)

- (a) Find the eigenvalue and Eigen functions of the sturm liouville problem in 1b (ii) above. [15 marks]
- (b) Define orthonormal functions F_1 and F_2 on an interval (a, b) then determine the orthogorality of two functions x^3 and x^4 on the interval $[-1, 1]$ [5 marks]

QUESTION THREE (20 MARKS)

(a) Show that the functions $f(x) = -6x + 2$, $f_2(x) = 6x^2 - 6x + 1$ and $f_3(x) = x - 1$ are orthonormal on $[0, 1]$ and obtain their orthonormal set [14 marks]

(b) Consider a system of two first order equations of the form

$$\frac{dx}{dt} = f_1(x, y)$$

$$\frac{dy}{dt} = f_2(x, y)$$

Explain in relation to the system

- (i) An autonomous system [1 mark]
- (ii) Phase plane [1 mark]
- (iii) Trajectory or the orbit [1 mark]
- (iv) Critical point [2 marks]

QUESTION FOUR (20 MARKS)

(a) Consider the system of equations

$$\frac{dx}{dt} = x - y + xy$$

$$\frac{dy}{dt} = 3x - 2y - xy$$

- (i) Verify that the point $(0, 0)$ is a critical point [2 marks]
 - (ii) Discuss the type and stability of the critical point $(0, 0)$ [10 marks]
- (b)
- (i) Explain the meaning of a regular SLP (Sturm Liouville Problem) [4 marks]
 - (ii) State four properties of regular Sturm Liouville problems [4 marks]