**CHUKA UNIVERSITY**

# UNIVERSITY EXAMINATIONS

**EXAMINATION FOR THE AWARD OF DEGREE OF MASTERS OF SCIENCE IN PURE MATHEMATICS**

**MATH 809: COMPLEX ANALYSIS I**

**STREAMS: BED (ARTS)** **TIME: 3 HOURS**

**DAY/DATE: TUESDAY 03/12/2019** **11.30 AM – 2.30 PM**

**INSTRUCTIONS:**

- Answer any three questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- This is a closed book exam, no reference materials are allowed in the examination room
- There will be NO use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

**QUESTION ONE (20 MARKS)**

(a) Explain the concept of conformal mapping. [3 marks]

(b) Find the image of the square $\pm 1 \pm i$ under the transformation $L(z) = 3z - 5 + 2i$. Show that all angles are maintained both in magnitude and sense. [11 marks]

(c) Show that $cot^{-1}(z) = \frac{1}{2i}\, In\left(\frac{z+i}{z-i}\right)$ and hence determine the principal value of z for which $cotz = 2i$ [6 marks]

**QUESTION TWO (20 MARKS)**

(a) Explain what is meant by a linear fractional transformation. [3 marks]

(b) Find a linear fractional transformation which maps the vertices $1+i, -i, 2-i$ of a triangle A on the z plane into the points 0,1,1 on the w-plane. [5 marks]

(c) Use the Beta function to show that $\int_0^\infty \frac{x^3}{1+x^6} dx = \frac{\pi}{3\sqrt{3}}$ [4 marks]

(d) Find general expression for $\Gamma\left(k+\frac{1}{2}\right)$ for $k = 0,1,2,...$ [4 marks]

(e) Let $f(z) = \frac{(z^2+4)^3}{(z^2+2z+2)^5}$, use the argument theorem to evaluate $\int_c \frac{f'(z)}{f(z)} dz$ where C is the rectangle enclosed by the line $y = 2.7, x = 3, y = 0$ $and$ $x = -2$ [4 marks]

**QUESTION THREE (20 MARKS)**

(a) State without proof Rouches' Theorem. [2 marks]

(b) Determine the number of roots for function $F(z) = 3z^6 - 412z^2 + 5z + 3$ inside region $2 \leq |z| \leq 4$. [4 marks]

(c) Prove that the function $\int_0^\infty t^4 e^{-2zt} dt$ is analytic for all the points z for which $Rez > 0$. Find an analytic continuation of f(z) into $Rez < 0$. [6 marks]

(d) Explain the maximum and minimum modulus theorem. [4 marks]

(e) Expand $f(z) = \frac{1}{(3+3)(z+1)}$ in a Laurent series valid for $0 < |z+1| < 2$. [4 marks]

**QUESTION FOUR (20 MARKS)**

(a) State without proof Hadamard Theorem. [2 marks]

(b) State and prove the Poissons integral formular for a circle. [10 marks]

(c) Evaluate the following $\int_c \frac{Cos\pi z}{(z+i)^4} dz, c: |z| = 2$ [4 marks]

(d) Prove that if F(Z) is analytic inside and on the boundary C of a simply connected region R and a is any point inside the curve C then $\int_C \frac{F(Z)}{Z-a} dz = 2\pi i\, F$ [4 marks]

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