CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF EDUCATION (ARTS), BACHELOR OF SCIENCE (GENERAL) AND BACHELOR OF ARTS (ECON STATS AND MATHS)

MATH 403: ORDINARY DIFFERENTIAL EQUATIONS II

STREAMS: B.ED (SC), BED (ARTS), BSC (GEN), BA (ECON, STATS &

TIME: 2 HOURS

DAY/DATE: TUESDAY 09/04/2019 **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS:

- **Answer question ONE (compulsory) and any other TWO questions**

QUESTION ONE (COMPUKLSORY) (30 MARKS)

(a) Rewrite the differential equation as a system of fist order equation in matrix form.

$y''' + 5 + y'' + 3y = 0$ (4 marks)

(b) Find the singular points of the Legendre equation and classify then

$(1-x^2)y'' - 2xy' + p(p+1)y = 0$ where p is a real number. (6 marks)

(c) The eigenvalues and corresponding eigenvectors are given as $\lambda_1 = 3, \vec{v}^{1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\lambda_2 = -1 \ \vec{v}^{2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Write down the general solutions of the system of differential equations. (3 marks)

(d) (i) State the existence theorem of differential equations at an ordinary point. (1 mark)

(ii) Show that $x=2$ is an ordinary point to the differential equation $x^2 y'' - x(x+3)y' + (x+3)y = 0$ and write the nature of its power series solution. (4 marks)

(e) Obtain the Legendre polynomials $P_4(x)$ from Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n$$ (7 marks)

(f) Solve the system of differential equations using matrix method. (5 marks)

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

QUESTION TWO (20 MARKS)

(a) Consider the system of differential equations

$$\frac{dx_1}{dt} = 2x_1 - x_2 - x_3$$

$$\frac{dx_2}{dt} = -x_1 + 2x_2 - x_3$$

$$\frac{dx_3}{dt} = -x_1 - x_2 + 2x_3$$

(i) Express the system in matrix form. (1 mark)

(ii) Find the general solutions to the system of differential equations. (7 marks)

(iii) Find the particular solution given the initial value $x_1(0)=0, x_2(0)=0$ and $x_3(0)=5$

(b) Find the series solution to the differential equation $y'' + 4y = 0$ at $x_0 = 0$. (9 marks)

QUESTION THREE (20 MARKS)

(a) Use the elimination method to solve the system. (11 marks)

$$Dx+(D+2)y=0$$

$$(D-3)x-2y=0$$

(b) (i) Define the Gamma functions and show that $\sqrt{x+1} = x\sqrt{x}$ (6 marks)

(ii) Show that $\sqrt{1} = 1$ (3 marks)

QUESTION FOUR (20 MARKS)

(a) Convert the IVP $y'''=t+2y+3y''$ with $y(0)=1, y'(0)=2$ and $y''(0)=3$ into a system of first order differential equation. (4 marks)

(b) Solve the initial value problem (10 marks)

$$\tilde{x}'(t)=\begin{bmatrix}1 & 2 & -1\\ 1 & 0 & 1\\ 4 & -4 & 5\end{bmatrix}\check{x}(t) \qquad x(0)=\begin{bmatrix}-1\\ 0\\ 0\end{bmatrix}$$

(c) Verify that $\left[\begin{pmatrix}e^{2t}\\ e^{2t}\\ e^{2t}\end{pmatrix}, \begin{pmatrix}-e^{-t}0\\ e^{-t}\end{pmatrix}, \begin{pmatrix}-e^{-t}\\ e^{-t}\\ 0\end{pmatrix}\right]$ is a fundamental set for the system (6 marks)

$$x'(t)=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}x(t)$$

QUESTION FIVE (20 MARKS)

(a) Solve the initial value problem

$$\frac{dx}{dt}=x+2y \qquad \text{With } x(0)=6 \text{ and } y(0)=4$$

(10 marks)

$$\frac{dy}{dt}=3x+2y$$

(b) Consider the Bessel's equation of order V.

$$x^2 y'' + x y' + \left(x^2 - v^2\right) y = 0$$

(i) Show that $x=0$ is a regular singular point to the Bessels equation. (3 marks)

(ii) Find the indicial equation of the Bessel's equation and state its significance.

(7 marks)
