CHUKA   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND STATISTICS, BACHELOR OF SCIENCE MATHEMATICS, BACHELOR OF EDUCATION SCIENCE, BACHELOR OF ARTS ECONOMICS AND MATHEMATICS

MATH 341: SAMPLING METHODS 1

STREAMS: B.ED SC, BSC MATH, BA ECON MATH, BASC ECON STAT

TIME: 2 HOURS

DAY/DATE: TUESDAY 16/04/2019   **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS:
- **Answer question 1 and any other 2 questions**

QUESTION 1 (30 MARKS)

(a)   State 4 sources of errors in sample surveys.   (4 marks)

(b)   State 3 principles of sampling theory and explain what is meant by each principle.
                                                                          (6 marks)

(c)   A population is found to have 12 observations. If samples of size 5 were taken from the population, state the number of samples that can be found if

        (i)   Sampling is done without replacement.   (1 mark)
        (ii)   Sampling is done with replacement.   (1 mark)
        (iii)   If the standard errors for the sample means in i and ii were computed, state one
                which would be more precise as an estimator of the standard deviation.
Support   your answer.
        (3 marks)

(d)   Consider a random sample of size n which has to be drawn from a finite population of size N. If simple random sampling method is used where sampling is done without replacement. Show that the probability of selecting a unit in the second draw is equal to selecting it in any other draw.   (5 marks)

(e)   The following data was extracted from a demographic journal.

Stratum	1	2	3	4	5
Stratum size	90	30	60	120	100
Stratum mean	6	10	12	8	
Stratum variance	2.25	3.24	3.61	2.89	1.96

(i)   If the entire population mean is 9.8, determine the missing stratum mean and hence the population standard deviation.

(ii)   For a stratified random sample of 162 elements, determine the sample size of the 4th stratum under Neyman allocation.

(f)   Explain clearly how sampling is carried out using systematic sampling procedure.
                                                                    (2 marks)

QUESTION 2 (20 MARKS)

(a)   Show that the stratified sample mean $(\acute{y}_{str})$ is an unbiased estimator of the population mean.   (5 marks)

(b)   A stratified population was found to have 5 strata. The stratum sizes, means and variances were displayed as shown.

Stratum	Size	Mean	Variance
A	41	9.1	1.96
B	74	11.2	1.13
C	117	7.3	1.31
D	45	9.6	1.74
E	98	6.9	2.03

(i)   Calculate the overall population mean and population variance.   (10 marks)

(ii)   For a stratified random sample of size 80, determine the appropriate stratum sample sizes under proportional allocations.   (5 marks)

QUESTION THREE (20 MARKS)

(a) With usual notations, show that when n=2, which Des Raj's estimator of the population total $\widehat{Y}_D = \frac{1}{2}(Z_1 + Z_2)$ where $Z_1 = \frac{y1}{p1} \wedge Z_2 + y_1 + y_2(\frac{1-p1}{p2})$ is an unbiased estimator of the population total. (5 marks)

(b) A survey was conducted to the yield of wheat in Nakuru county. A sample of 10 farms from a total of 100 was taken with a probability proportional to the area under wheat crop with replacement method. The total areas under wheat (x) was 484.5 hectares. The sample selected using Lahiri's method was

Areas under wheat crop x_i	4.8	4.1	1.3	5.2	6.9	2.0	6.3	5.2	4.2	6.0
Yield of crop y_i	22	19	6	25	54	4	45	28	29	44

Estimate the average yield per farm with its standard error. (10 marks)

(c) A sub-county has 10 secondary schools having 150,230,345,125,420,260,124,326,282 and 198 students. Explain clearly how you would select a sample of 4 schools in the sub-county with probability proportional to the number of students in the schools using Lahiri's method if sampling is done with replacement. (5 marks)

QUESTION FOUR (20 MARKS)

(a) Given that $V_{rand} = \left(\frac{N-n}{Nn}\right)S^2, V_{\dot{c}} = \frac{\Sigma_{i=1}^{k} w_i s_i^2}{n} - \frac{\Sigma_i^{k}=1 w_i s_i^2}{N}$ and

$$V_{opt} = \dot{c} \frac{\Sigma_{i=1}^{k} w_i s_i \dot{c}^2}{\dot{c}}$$

Assuming that the terms in $\frac{1}{N_i}$ are ignored relative to unity, show

that $V_{opt} \le V_{\dot{c}} \le V_{rand}$ where the optimum allocation for fixed n. (7 marks)

(b) From past surveys, the variance of the amount of money spent on "research and development" and the cost per interview are known. The firms involved are classified as "small", "medium" or "large". A budget for the study of these firms was set as sh 20,000, the set up costs were negligible, the following information was gathered.

Firm classification	Number of firms	Variance of the money spent ($)	Cost per interview ($)
Large	500	10000	25
Medium	1000	900	16
Small	7000	400	9

(i)   Design a proportional allocation sample   (8 marks)

(ii)   Design an optimal allocation sample   (5 marks)

QUESTION 5 (20 MARKS)

(a)   A population consists of N=n M units. Explain how you would obtain a systematic sample of size n from this population.   (5 marks)

(b)   Under what conditions would you prefer systematic sampling to simple random sampling?   (2 marks)

(c)   Consider the following items in a population 15, 17, 14, 18, 15, 16. Suppose a sample of size 5 is selected by simple random sampling without replacement. By enumerating all the possible samples, show that;

(i)   The sample means is an unbiased estimator of the population mean. (5 marks)

(ii)   The sample variance is an unbiased estimator of the population variance.
                                                                                (4 marks)

(iii)   In simple random sampling the variance of sample means satisfies the formula
$$Var(\acute{y}) = \frac{N-n}{N} \frac{s^2}{n}$$
                                                                                (4 marks)