CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE/ARTS; BACHELORS OF SCIENCE GENERAL, BACHELORS OF ARTS(MATHS-ECONS)

MATH 304: COMPLEX ANALYSIS

STREAMS: ````AS ABOVE```` Y3S2 **TIME: 2 HOURS**

DAY/DATE: FRIDAY 12/04/2019 **11.30 A.M – 1.30 P.M**

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

a) Determine the four roots of the complex number $z = 2 - 2i$ (4 marks)

b) Evaluate the limits of the following functions

i. $\lim_{z \to \pi} \frac{\sin z}{\pi - z}$ (2 marks)

ii. $\lim_{z \to i} \frac{e^{\pi z} + 1}{z^2 + 1}$ (2 marks)

c) Solve the equation $\cos z = -3$ (4 marks)

d) Evaluate the following complex integrals

i. $\int \frac{\sin z + \cos z}{(z-\pi)^2(z+4)}dz$ inside the circle C is the circle $|z| = 3.5$ (5 marks)

ii. $\int \frac{e^{2iz}}{1+4z^2}dz$ inside the region bounded by the circle $|z| = 1$ (5 marks)

e) Find the locus of the points z in the complex plane such that $|z-2i| = |z+2|$ (3 marks)

f) Show that the function $f(x,y) = 2y^3 - 6x^2y + 4x^2 - 7xy - 4y^2 + 3x + 4y - 4$ is harmonic and find its harmonic conjugate $g(x,y)$ such that $h(z) = f(z) + ig(z)$ (5 marks)

QUESTION TWO (20 MARKS)

a) i)Given $w = \log z$, where $z = x + iy$ show that

$\text{Re}(w) = \frac{1}{2}\log(x^2 + y^2)$ and $\text{Im}(z) = \tan^{-1}(\frac{y}{x}) + 2k\pi; k \in Z$ (4 marks)

(ii)Taking the principal value of w, show that the function $w = \log z$ satisfies the Cauchy-Riemann equations (4 marks)

(iii) Express $\log(1+i) + \log(1-i)$ in the form of the real and imaginary parts(4 marks)

b) Show that the function $f(z) = x^2 - y^2 + i(2xy + y)$ is differentiable and find its derivative (4 marks)

c) Prove that if $u(r,\theta) = u(r\cos\theta, r\sin\theta)$ is analytic, then $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{dr} + \frac{1}{r^2}\frac{\partial u}{\partial r} = 0$. (Assume the polar form of Cauchy-Riemann equations) (4 marks)

QUESTION THREE(20 MARKS)

a) Without using the cauchy theorem, evaluate $\int_c \bar{z}\, dz$ from $z=0$ to $z=4+i$ along the curve $z=0$ to $z=2i$ and the line from z ¿$2i$ to $4+2i$ (5 marks)

b) Evaluate the integral $\frac{1}{2\pi i}\oint_c \frac{e^{zt}}{z^2+1}dz$ when $t > 0$ and C is the circle $|z-i| = 1$ (5 marks)

c) State without proof the residue theorem and use it to evaluate the integral

$$\oint_c \frac{z^2+5}{z^2(z+3)^3(z^2+4)}dz \qquad C; \ |z-2i|=3$$

(10 marks)

QUESTION FOUR (20 MARKS)

a) Prove that if a limit of a complex valued function exists, then it is unique (6 marks)

b) Determine the region and the radius of convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}z^{2n-1}}{(2n-1)!}$ (6 marks)

c) Show that $\sin^{-1} z = -i\log[iz+(1-z^2)^{\frac{1}{2}}$ (4marks)

d) Verify that the function $f(z) = x^3\sin y - 3xy^2 + i(3x^2y - y^3\cos 2x)$ is not analytic
. (4 marks)

QUESTION FIVE (20 MARKS)

a) Differentiate isolated, non-isolated singularities and essential singularity using an appropriate example in each case. (6 marks)

b) Expand the function $f(z)=\frac{1}{z^2-4z+3}$ about the circle $|z-4|<1$. (7 marks)

c) By expanding the function $f(z)=\frac{1}{1+z^2}$ about $z=0$ and $z=i$, explain the difference between the two expansions (7 marks)
