CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND STATISTICS, BACHELOR OF ARTS IN ECONOMICS AND SOCIOLOGY, BACHELOR OF ARTS IN ECONOMICS AND MATHEMATICS AND BACHELOR OF ARTS IN ECONOMICS AND HISTORY

ECON 232: MATHEMATICS FOR ECONOMIST I

STREAMS: AS ABOVE **TIME: 2 HOURS**

DAY/DATE: TUESDAY 10/12/2019 **11.30 AM – 1.30 PM**

INSTRUCTIONS:

Answer Question One and any other Two Questions from the remaining

QUESTION ONE

Suppose you are in charge of economic planning for a region characterized by three production sectors; grain production, automobiles and electrical power. Last year, the grain sector consumed 3 units of its gross output in its own production process and delivered 5 units to automobiles and 10 units to final consumers. The automobiles sector delivered 4 units to grain, 2 units to electrical power and 6 units to final consumers and it used 2 units in its own production. Electrical power used 3 units of electricity of electricity in its own production, it delivered 20 units to automobiles, 5 units to grain and 8 units to grain, 10 units to automobiles and 5 units to electrical power. In addition, 4 units of labour were employed by final consumers.

(i) Set up the input – output (I-O) table for this economic region. [10 marks]

(ii) Describe the impact of a three unit decrease in the gross output of the grain sector on all sectors of your region. Construct the input output table for the new situation. [10 marks]

(iii) Describe the impact of an increase in final demand for automobiles of 2 units (assume the original input – output table). Construct the input table for the new situation. [10 marks]

QUESTION TWO

(i) Given the following equation $y^3 + 3xy^2 + 3x^2y + x^3 - 8y + 4x = 0$. Determine $^{dy}/_{dx}$ at the point $(1, 1)$ [5 marks]

(ii) Let $Z = f(x, y) = x^3 - x^2y + xy^2 - y^3 and\ y = g(x) = 3x - 8$. Find $^{dz}/_{dx}$ the total derivative using any method. Express the answers as a function of x only. Do not develop the expression. [5 marks]

(iii) Let $z = f(x, y) = \sqrt{2x^2} + xy + y^2$. Use the differential dz to estimate the change in z i.e. Δz when moving from $(x, y) = (2, -2)\ to\ (1.9, -1.9)$. Compare the results with the actual value of ΔZ. [5 marks]

(iv) You are asked to determine the (K, L) settings that will minimize the total cost TC, with the constraint that the production levels Q must remain at a constant value of $Q_0 = 32$. The model for total cost is TC= 9L+72K and the production function uses the standard Cobb-Douglas model, with $A = 1, \alpha = {}^1/_4, \beta = {}^1/_2$ [5 marks]

QUESTION THREE

(i) Let $Z = f(x, y) = 2x^2 - 6x + 12y + 18$. Determine the nature i.e. min, max or saddle of the critical point (-1,2). Make sure to do all the work necessary to justify your answers. [5 marks]

(ii) Let $Z = f(x, y) = 4x^2 + 3y^2 - 12xy + 144x - 120y + 16$. Determine the (x, y) coordinates of the critical points. Do not determine if these are a max, min or saddle point. [5 marks]

(iii) Let $Q = f(K, L) = 4K^{1/2} L^{1/2}$ be the rule for a Cobb-Douglas production function where K corresponds to input capital, L correspond to input labour and Q correspond to output production. Assuming $Q = Q_0 = 8$. Determine the Marginal Rate of Technical Substitution (MRS). [5 marks]

(iv) If $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}, show\ that\ f'(x) = \frac{x-1}{2x\sqrt{x}} = 0$ [5 marks]

QUESTION FOUR

(i) Find the inverse A^{-1} for the following matrix given below

$$A = \begin{bmatrix} 24 & 15 \\ 8 & 7 \end{bmatrix}$$ [5 marks]

(ii) Use matrix inversion to solve the following systems of linear equations

$4x_1 + 3x_2 = 28$
$2x_1 + 5x_2 = 42$ [5 marks]

(iii) Use Cramer's rule to solve for x, y and λ given the following first order condition for constrained optimization

$\frac{\partial TC}{\partial x} = 16x - y - \lambda = 0$

$\frac{\partial TC}{\partial y} = 24y - x - \lambda = 0$

$\frac{\partial TC}{\partial \lambda} = 42 - x - y = 0$ [10 marks]
