CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DOCTOR OF PHILOSOPHY IN APPLIED STATISTICS

MATH 956: DESIGN AND ANALYSIS OF MIXTURE EXPERIMENTS

STREAMS: PhD (APPLIED STATS) **TIME: 3 HOURS**

DAY/DATE: WEDNESDAY 14/08/2019 **2.30 P.M - 5.30 P.M.**

INSTRUCTIONS:

- **Answer ANY THREE Questions**.
- **Show ALL your Working Clearly**

QUESTION ONE (20 MARKS)

An experimenter is trying to optimize the formulation of automotive clear coat paint. These are complex compounds that have very specific performance requirements. Specifically, the customer wants the Knoop hardness to exceed 25. The clear coat is a three component mixture, consisting of a monomer (x_1) , a crosslinker (x_2) and a resin (x_3). There are constraints on the component proportions:

$x_1 + x_2 + x_3 = 100$

$5 \leq x_1 \leq 25$

$25 \leq x_2 \leq 40$

$50 \leq x_3 \leq 70$

The region of interest is not a simplex because of the constraints and therefore a D-optimal design is used. Assume that the response is likely to be modelled with a quadratic mixture model. Design Expert was used to generate a 14-run D-optimal design below:

Standard Order	**Run**	**Monomer (x_1)**	**Crosslinker (x_2)**	**Resin (x_3)**	**Hardness (y)**
1	2	17.50	32.50	50.00	29
2	1	10.00	40.00	50.00	26
3	4	15.00	25.00	60.00	17
4	13	25.00	25.00	50.00	28

5	7	5.00	25.00	70.00	35
6	3	5.00	32.50	62.50	31
7	6	11.25	32.50	56.25	21
8	11	5.00	40.00	55.00	20
9	10	18.13	28.75	53.13	29
10	14	8.13	28.75	63.13	25
11	12	25.00	25.00	50.00	19
12	9	15.00	25.00	60.00	14
13	5	10.00	40.00	50.00	30
14	8	5.00	25.00	70.00	23

The ANOVA table for the Mixture Quadratic Model was:

Component	**Coefficient Estimate**	**DF**	**Standard Error**
A - Monomer	32.81	1	3.36
B - Crosslinker	16.40	1	7.68
C - Resin	29.45	1	3.36
AB	44.42	1	25.31
AC	-44.04	1	15.94
BC	13.80	1	23.32

Source	***Sum of Squares***	***DF***	***Mean Square***	***F value***	***Prob > F***
Model	*279.73*	-	-	-	*0.1329*
Residual	*188.63*	*8*	-		
Lack of Fit	*63.63*	-	-	-	*0.7354*
Pure Error	*125.00*	*4*	-		
Cor Total	*468.36*	*13*			

R^2 = 0.5973 R^2_{Adj} = 0.3455 R^2_{Pred} =-0.3635 *Adeq Precision* = 4.975 *C.V.* =19.59 *PRESS*= 638.60

a) Draw a 2-dimensional diagram to represent the constrained experimental region for the paint formulation problem.

b) Six runs were required to fit the quadratic mixture model, justify the purpose of the extra 8 runs.

c) Fill in the Gaps in the table

d) Write the final equation of the model and using any of the available information in the tables discuss the model.

e) Advise the experimenter on how he would optimize the formulation having in mind the customer's requirements and the results obtained above.

QUESTION TWO (20 MARKS)

A detergent company is trying to produce a new cleaning solvent for circuit boards. It anticipates that a combination of three standard solvents will work as well as the specialty solvent currently in use, but beyond knowing that they want each of the three solvents to be at least 10% of the combination, they don't know how much of each to use. Use this information to advice with reasons the company on:

a) The choice of the proper model to approximate the surface over the region of interest

b) The testing of the adequacy of the model in representing the response surface

c) A suitable design for collecting the observations, fitting the model and testing the adequacy of the fit model

d) Optimization of the response

QUESTION THREE (20 MARKS)

An experimenter set up an experiment involving three mixture components. There were no constraints on the mixture proportions. The following results were observed:

Design Point	**Component Proportions**			**Observed**
	x_1	x_2	x_3	y_u
1	0	0	1	10.3, 13.4, 11.1
2	0.5	0.5	0	16.4, 15.0, 16.1, 14.9
3	0	0.5	0.5	11.6, 13.2, 12.5, 14.0
4	1	0	0	14.0, 15.9, 14.8
5	0.5	0	0.5	17.2, 16.3, 16.9, 15.8
6	0	1	0	15.6, 18.3, 19.4

a) Describe the type of design used by the experimenter

b) Fit a quadratic mixture model to the data

c) Calculate the estimates of the standard errors and place them in parentheses directly below the corresponding parameter estimate in the fitted model. Comment on the fitted model.

d) Taking the point $\mathbf{x} = (1/3, 0, 2/3)'$
   (i) Obtain the estimate of the response at this point
   (ii) Obtain a 95% confidence interval for η at this point

QUESTION FOUR (20 MARKS)

An experiment was set involving compressive strength of concrete. 54 experiments in a complete random design for two processes (Z_i); i = 1,2 and three mixture variables: j = 1,2,3 were carried out.

Formulation No.	**1**		**2**		**3**	
	R_1	**R_2**	**R_1**	**R_2**	**R_1**	**R_2**
$z_1 = -1$; $z_2 = -1$	16.40	11.12	6.41	12.15	6.52	07.12
$z_1 = 0$; $z_2 = -1$	16.65	17.68	21.10	10.38	10.88	10.70
$z_1 = 1$; $z_2 = -1$	15.59	11.02	9.21	12.29	15.23	10.63
$z_1 = -1$; $z_2 = 0$	5.17	4.57	3.86	4.04	3.54	0.00
$z_1 = 0$; $z_2 = 0$	7.72	5.35	3.01	4.43	3.97	4.61
$z_1 = 1$; $z_2 = 0$	11.05	8.89	5.46	6.02	6.09	4.99
$z_1 = -1$; $z_2 = 1$	3.15	0.00	0.00	0.00	0.00	0.00
$z_1 = 0$; $z_2 = 1$	2.94	9.53	2.59	8.15	0.00	10.66
$z_1 = 1$; $z_2 = 1$	4.00	8.61	2.69	10.91	0.00	15.41

Each mixture hand 2 replicates R_1 and R_2. Assuming that the model fitted was a combination of a linear mean model for the process and a quadratic mixture model for the mixture:

a) Formulate the mixture-process model for this experiment.

b) Illustrate a general structure of a suitable design for this experiment

c) Explain the need for replication and randomization in this experiment

d) Different optimal designs can be obtained using design software for use in setting up this experiment. Explain how the most optimal design can be determined

e) Explain some of the considerations that need to be taken when setting up such an experiment

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