CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS

MATH 932: MATHEMATICAL EDIDEMIOLOGY

STREAMS: PhD **TIME: 3 HOURS**

DAY/DATE: MONDAY 12/08/2019 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS:

- **Answer ALL questions.**

QUESTION ONE

(a) Show that the logistic growth model for a single specific population is given as

$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right)$, hence solve the equation where

$k =$ is the carrying capacity $N(t) -$ number of individuals in a population at time t and r is the growth rate. (10 marks)

(b) Explain the following types of interactions between two species.

(i) Neutralism (1 mark)
(ii) Mutual inhibition competition (1 mark)
(iii) Amensalism (1 mark)
(iv) Predation/parasitism (1 mark)
(v) Mutualism/symbiosis (1 mark)

QUESTION TWO

(a) Give 3 assumptions of the predator prey model, Hence derive the predator pre model. (3 marks)

(b) Solve the predator prey model developed in (a) above and use the solution to show that in the absence of the predators, the prey grows exponentially and in the absence of the prey the predator population dries out. (7 marks)

(c) Given the system of equations below

$$\frac{dx}{dt} = x - y + xy$$

$\frac{dy}{dt} = 3x - 2y - xy$, whwre $x -$ prey and $y -$ predator. Verify that (0, 0) is a critical point and also show that the system is almost linear and discuss the type and stability of the critical point (0, 0) (5 marks)

QUESTION THREE

(a) The differential equations for a competing model system is given as

$$\frac{dx}{dt} = r_1 x - \alpha_1 xy$$

$$\frac{dy}{dt} = r_2 y - \alpha_2 xy$$

Where α_1 and α_2 are two constants.

(i) Obtain the solution for x in the absence of the second species i.e. when $y = 0$

(ii) Obtain the non-trivial steady state point of the system (x^x, y^x) (4 marks)

(iii) Show that the equilibrium density of one species depends upon the proportional growth and the coefficient of inter-specific coefficient of the other species. (4 marks)

(iv) Examine the stability of the steady state (x^x, y^A) using the perturbation technique. (5 marks)

QUESTION FOUR

(a) Given the epidemic model

$\frac{dx}{dT} = -x(n + 1 - x),$ where $x = n\ at$ T=0. Where $x(t)$ is the number of susceptible and $T = \beta t.$

(i) Use the substitution $x = \frac{1}{u}$ to solve the equation. (6 marks)

If the contact rate β is 0.001 and the number od susceptible (n) is 2,000. Initially, determine

(ii) The number of susceptible left after 3 weeks. (2 marks)

(iii) The density of susceptible when the rate of appearance of new cases is maximum. (2 marks)

(iv) The time in weeks at which the rate of appearance of new cases is maximum. (2 marks)

(v) The maximum rate of appearance of new cases. (2 marks)

(v) The epidemic curve. (1 mark)
