**CHUKA**  **UNIVERSITY**



**UNIVERSITY EXAMINATIONS**

**EXAMINATION FOR THE AWARD OF DOCTOR OF PHILOSOPHY IN APPLIED MATHEMATICS**

**MATH 925: MATHEMATICAL BIOLOGY**

**STREAMS: PhD** **TIME: 3 HOURS**

**DAY/DATE: TUESDAY 13/8/2019** **2.30 P.M. – 5.30 P.M**

**INSTRUCTIONS: ANSWER ALL QUESTIONS**

**QUESTION ONE**

(a) Given the system $S(\bar{X}, R)$ denoting a spherical region in the state space with centre at $\bar{X}$ and radius R. define the followng terms

(i) An equilibrium point $\bar{x}$ [2 marks]

(ii) An asymptotically stable equilibrium point $\bar{x}$ [2 marks]

(iii) A marginally stable equilibrium point $\bar{x}$ [2 marks]

(iv) An unstable equilibrium point $\bar{x}$ [2 marks]

(b) Given the system

$\dot{x}(t) = x(t)$ and $\dot{y}(t) = -k\, y(t)$

Find the solution of the system and determine the stability of the system when $k = 1, 2$ and $-1$ [7 marks]

**QUESTION TWO**

(a) Given the system $\dot{x}_1 = k_1(1 - \varepsilon x_1 - x_2)x_1, 0 < x \ll 1$

$\dot{x}_2 = k_2(1 - x_1)x_2$

Show that the system has three equilibrium points

$(0,0)$, $(1,1-\varepsilon)$ and $\left(\frac{1}{q},0\right)$ and linearize the system about the equilibrium points and determine the nature of the equilibrium points. [8 marks]

(b) (i) State the Liapunor theorem [3 marks]

(ii) Given the system

$\dot{x}_1(t) = x_2(t), \dot{x}_2(t) = x_1(t) - x_2(t)$, show that the equilibrium point $(0,0)$ is stable by liapunor theorem. [4 marks]

**QUESTION THREE**

(a) Determine the invariant set for the system

$\dot{x}_1 = x_2 + x_1(1 - x_1^2 - x_2^2)$

$\dot{x}_2 = -x_1 + x_2(1 - x_1^2 - x_2^2)$ [3 marks]

(b) Determine the birfurcation point for the system [4 marks]

$\dot{x}_1 = x_2$

$\dot{x}_2 = mx_1$

(c) Find the bifurcation points of the system

$\dot{x}_1 = -kx_1 + x_2$

$\dot{x}_2 = -kx_1 - 3x_2$ [8 marks]

**QUESTION FOUR**

(a) Show that the system

$\dot{x}_1 = x_2$

$\dot{x}_2 = \mu x_2 - x_1^2 - x_2$, has two equilibrium points $(0,0)$ and $(\mu,0)$ and a transcritical bifurcation

(b) Explain the 3 main types of interactions in population dynamics. [3 marks]

(c) Explain the main assumptions in the predator prey model [3 marks]

(d) Explain the SIR epidemic model [2 marks]

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