CHUKA


## ECON 131: INTRODUCTION TO MATHEMATICS FOR ECONOMISTS

STREAMS: BSC (ECON \& STATS/ECON \& SOCI/ECON \& MATHS/ECON \& HIST (Y1S1)

TIME: 2 HOURS
DAY/DATE: MONDAY 09/12/2019
8.30 A.M. - 10.30 A.M.

INSTRUCTIONS: Answer question ONE and any other TWO questions. Show your workings clearly

## QUESTION ONE (30 MARKS)

(a) Differentiate between the following terms:
(i) Behavioural and technical functions
(ii) Endogenous and Exogenous variables
(b) A thousand people took part in a survey to reveal which newspaper A, B or C; they had read on a certain day. The responses showed that 420 had read A, 316 had read B, and 160 had read C. Of these responses, 116 had read both A and B, 100 had read A and C, 30 had read B and C and 16 had read all three papers
(i) How many had read A but not B?
(ii) How many had read both A and B and not C ? [2 marks]
(iii) How many had read neither $\mathrm{A}, \mathrm{B}$ and C
(c) Evaluate
(i) $4 \log _{4} 3+2 \log _{3}\left(\frac{1}{9}\right)$
(ii) Simplify $\left(x^{-5}\right)\left(x^{8}\right) \div x^{3}$
[1 mark]
(d) A national income model is given by
$Y=C+I+G$
Where $\mathrm{C}, I$ and G are consumption, investment and government expenditure components given by:
$C=a+b y$
$I=0.1 Y$
$G=250$

## Required:

(i) Find equilibrium value of income (Y)
(ii) What is the corresponding equilibrium investment?
(e) Find the following
(i) $\int\left(x^{3}+\sqrt{x}-x^{\frac{1}{2}}\right) \partial x$
(ii) $\quad \int_{2}^{5}(x+2)(x-1) \partial x$
(f) The average revenue and total cost for a firm are given by:
$A R=31 / 2-1 / 2 Q$
$T C=\frac{1}{20} Q^{3}-\frac{3}{10} Q^{2}+2 Q+1$

Find:
(i) The output and price levels that will maximize profits.
(ii) The output level that will maximize total revenue
(iii) The total output level that will minimize average variable cost (AVC) marginal cost (MC)
(iv) The minimum AVC and MC
(g) Expand $(9+x)^{10}$

## QUESTION TWO (20 MARKS)

(a) Consider the following equations
$P Q=10$
$Q=-2 \sqrt{5}+2 P$
(i) Which of the two represents supply and which represents demand function?
(ii) Determine the equilibrium price and quantity
(iii) Graph your results
(b) Find the derivatives of
(i) $\quad Y=\left(3 x^{2}+2 x+1\right)^{-3 / 4}$
(ii) $\quad Y=\left(x^{2}\right) \sqrt{\left(x^{2}+2 x^{2}\right)}$
(iv) $\quad Y=\frac{\left(x^{2}+1\right)(x+5)}{(x+6)\left(x^{2}+x\right)}$
(b) Consider the following demand functions for two firms
$P=16-0.4 Q$
$P=\frac{1}{3} Q^{2}-16 \frac{1}{2} Q+230 \quad 0<Q<20$
In each case
(i) Write out the corresponding elasticity of demand
(ii) Determine price elasticity of demand at $P=6$ and $Q=20$
(iii) Comment on the magnitude and sign of your results

## QUESTION THREE (20 MARKS)

(a) (i) Find the points at which critical values for the following function occur and whether the function attains maximum or minimum value at such points [4 marks] $Y=\frac{1}{3} x^{3}+\frac{5}{2} x^{2}+6 x+23$
(ii) Consider the following demand and cost functions
$P=25-3 Q$
$T C=Q^{2}+6 Q$
If a per unit $\operatorname{tax} t$ is imposed on the output, determine
(I) The maximum profit
(II) The change in price
(III) The tax rate $t$ which will maximize total tax $T=t Q$
(IV) The maximum tax T
(b) Some two commodities how the following demand and supply functions:

$$
\begin{aligned}
& Q d 1=4-2 P_{1}+2 P_{2} \\
& Q d_{2}=6+2 P_{1}-2 P_{2} \\
& Q s_{1}=-3+P_{1} \\
& Q s_{2}=-2+2 P_{2}
\end{aligned}
$$

Determine the equilibrium values of prices and quantities for the two commodities using Cramer's rule
(c) The inter industry transactions for a four sector economy are given by:

|  | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 120 | 0 | 200 | 180 |
| 3 | 80 | 40 | 0 | 36 |
| 4 | 61 | 158 | 0 | 240 |
|  | 100 | 36 | 37 | 0 |

(i) Which of the sectors has purchased inputs from all the other sectors except sector 1 ?
[1 mark]
(ii) Which of the four sectors has purchased products from all the others except itself and sold its output to all the other except itself?
(iii) How much has sector 2 sold to itself?
(iv) If total output of sector 2 is 356 ; how much does the sector contributes to find demand?
(v) If total input purchased by sector 3 is 440 , how much value added did this sector use in its production process

## QUESTION FOUR

(a) The supply of cooking oil is given by the following function

$$
Q=-4+\frac{1}{2} P
$$

Where Q is the supply of cooking oil and P is its price
(i) Graph the function
(ii) What is the quantity of cooking oil supplied at zero price? [1 mark]
(iii) What happens to the supply for cooking oil as its price rises? [1 mark]
(iv) 'The supply of cooking oil increases as its price falls' is this statement correct?

Explain
[1 mark]
(b) An economy is defined by:
$\mathrm{Y}=\mathrm{C}+\mathrm{I}+\mathrm{G}+\mathrm{X}-\mathrm{M}$
$C=C+C_{o}+C_{1} Y$
$\mathrm{I}=\mathrm{I}_{\mathrm{o}}+\mathrm{I}_{1} \mathrm{Y}$
$\mathrm{M}=\mathrm{M}_{\mathrm{o}}+\mathrm{M}_{1} Y$
$\mathrm{G}=G_{o}$
$\mathrm{x}=\mathrm{X}_{\mathrm{o}}$
(i) Name the endogenous and the exogenous variable in the model [2 marks]
(ii) What is the difference between $C_{o}$ and $M_{0}$ on one hand, and $C_{1}$ and $M_{1}$ on the other hand?
(iii) What is the balance of trade position in the economy when
(a) $\quad X>M$
(b) $\quad X<M$
(c) $\quad X=M$
(iv) Find the value of $Y$ in terms of $C_{o}, C_{1}, I_{0}, I_{1}, M_{o}, M_{1}, X_{o}$ and $G_{o}$ and denote the value by $\bar{Y}$
(v) Find the value of C in terms of $\mathrm{C}_{\mathrm{o}}, \mathrm{C}_{\mathrm{I}}, \mathrm{I}_{\mathrm{o}}, \mathrm{I}_{1}, \mathrm{M}_{\mathrm{o}}, \mathrm{M}_{1}, \mathrm{X}_{\mathrm{o}}$ and $\mathrm{G}_{\mathrm{o}}$ and denote it by $\bar{C}$
(c) Solve $4 x^{2}-25=0$

