CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE/ARTS, BACHELORS OF SCIENCE, BACHELORS OF ARTS(MATHS-ECONS), BACHELORS OF SCIENCE(ECON STATS)

MATH 301: LINEAR ALGEBRA II

STREAMS: `` As above``

TIME: 2HRS

DAY/DATE: ……………………. ………………………..

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

a) Given that , find the eigenvalues of (5 marks)

b) Let A be an idempotent matrix (). Prove that det(A) is either 0 or 1 (3 mark)

c) Find the symmetric matrix that correspond to the following quadratic form

(2 marks)

d) Consider the basis U of spanned by the vectors , use the Gram Schmidt formula to find an orthonormal basis. (4 marks)

e) State how elementary row operations affect the determinant of a square matrix, hence or otherwise show that if two rows are the same the determinant is zero. (5 marks)

f) Verify whether or not the following statements are true. If they are give a proof. If not give a counter example.

i. If two n-square matrices are similar, then they have the same characteristic polynomial

ii. For a an n square matrix A to be diagonalizable, then it must to have n distinct eigenvalues (5 marks)

g) Determine whether the quadratic form is positive definite (2 marks

h) Find the minimal polynomial of the matrix (4 marks)

QUESTION TWO (20 MARKS)

a) Let be a bilinear form on defined by . Find

i. The matrix A of in the basis

ii. The matrix B of in the basis

iii. The change of basis matrix P from the basis to the basis and verify that . (12 marks)

b) Let A be the matrix

Apply diagonalization algorithm to obtain a matrix P such that (8 marks)

QUESTION THREE (20 MARKS)

a) Given thatdetermine the number and the sum of principal minors of order 1, 2 and 4. (7 marks)

b) Let

i. Find the characteristic polynomial of A. (3 marks)

ii. Find all the eigenvalues of A and their corresponding eigenvectors. (8 marks)

iii. Is A diagonalizable? If yes, Determine the matrices P and D such that such that D is diagonal. (2 mark)

QUESTION FOUR (20 MARKS)

a) State Cayley-Hamilton theorem for a linear operator and verify the theorem using a linear operator defined by (4 marks)

b) Let $V \subset R[x]$ be the polynomials of degree 2 or less. let $T: V \rightarrow V$ be the linear operator $T(p(x))$

$= (x + 1)p'(x)$. Find the minimal polynomial of T (6 marks)

c) Suppose that is the quadratic form associated with the symmetric bilinear form verify that when (3 marks)

d) Consider the quadratic form and the linear substitution and

i. Rewrite in matrix notation and find the matrix notation and find the matrix A representing (1 mark)

ii. Rewrite the linear substitution using matrix notation and find the matrix P corresponding to the substitution (2 marks)

iii. Write the quadratic form (2 marks)

iv. Verify part iii above using direct substitution (2 marks)

QUESTION FIVE (20 MARKS)

a) i. Define a complex inner product space. (3 marks)

ii. Let V be a complex inner product space, verify that (3 marks)

iii. Suppose, evaluate (3 marks)

b) Prove that a set of orthonormal vectors is linearly independent (3 marks)

c) Calculate the coordinate matrix of relative to the orthonormal basis determined by the vectors in with the inner product .

(4 marks

d) Find an orthonormal basis for the solution space to the orthonormal system of equations below

(4 marks)