CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF SCIENCE MATHEMATICS

MATH 204: ALGEBRAIC STRUCTURES

STREAMS: `` As above``

TIME: 2HRS

DAY/DATE: ……………………. ………………………..

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) Let *be a binary operation on the set of integers, defined by

for every . Determine whether or not * is

i. Commutative

ii. Associative

iii. Find an identity element with respect to * if it exists (4 marks)

b) Let denote the set of all 2x2 matrices with integer coefficients, whose determinant is one. Verify whether or not is a group under matrix multiplication .
(5 marks)

c) Given a group G, define the centre of G,(z(G)) and show that it is a normal subgroup of G (4 marks)

d) Prove that a group G is abelian, then the mapdefined by ,is a group homomorphism
(2 marks)

e) Suppose a,b and c are elements of an integral domain D such that ab=ac and . Prove that b=c (3 marks)

f) Verify whether or not the following statements are true about groups

i. A group of order 21 has a subgroup of order 10 (2 marks)

ii. Every cyclic group is abelian (2 marks)

g) The addition and part of the multiplication table for the ring R={a,b,c,d} are given below. Use the distributive laws to complete the multiplication table below
. (4 marks)

+	A	B	C	D
A	A	A	C	D
B	B	C	D	A
C	C	D	A	B
D	D	A	B	C

.	A	B	C	D
A	A	A	A	A
B	A	C		D
C	A		A	
D	A		A	C

h) Verify in each case whether or not the set I defined below is an ideal in the ring R where;

i. R is the ring of rational numbers and I is the se of all non-negative rational numbers (2 marks)

ii. R is the ring of polynomials with integer coefficients and I is the set of polynomials in R whose leading coefficient is even (2 marks)

QUESTION TWO (20 MARKS)

a) Let n be a positive integer. Define as and where denotes the remainder of division of k by n.

i. Show that is a group homomorphism (2 marks)

ii. Find ker (2 marks)

iii. Find the index (2 marks)

iv. Find all the homomorphisms if any exists (2 marks)

b) Consider the set .

i. Construct addition and multiplication tables for R using operations as defined in (4 marks)

ii. Show that R is a commutative ring with unity. (2 mars)

iii. Show that R a subring of (2 marks)

iv. Does R have zero divisors? (1 marks)

v. Is R a field? If yes illustrate each element with its inverse (3 mark)

QUESTION THREE (20 MARKS)

Construct the multiplication table for the group of symmetries of a square

List all the subgroups of . Which of these are normal subgroups? (20 marks)

QUESTION FOUR (20 MARKS)

a) Let U be a fixed non-empty set and R be the set of subsets of U with addition and multiplication defined by and . Verify whether or not is a ring. (6 marks)

b) Consider the ring and let I be the even coset in R i.e. r+R such that r is an even integer. Show that I is an ideal of R (7 marks)

c) Show that in any ring the zero element is unique (2 marks)

d) Let R be a ring such that every element satisfies the equation , prove that R is commutative (4 marks)

QUESTION FIVE (20 MARKS)

a) Let G be a group in which every element has order at most 2. Show that G is abelian (3 marks)

b) Show that in an abelian group G, the set of all elements with finite order in G is a subgroup of G. (5 marks)

c) Let G be the set of eight elements given by with multiplication given by,,, .

i. Construct a multiplication table for the group. (6 marks)

ii. Consider the cyclic group group . list all the distinct cosets of H in G. Is H a normal subgroup of G? (6marks)