CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE IN CHEMISTRY

CHEM 833: ADVANCED MATHEMATICS FOR CHEMISTS

STREAMS: MSC **TIME: 3 HOURS**

DAY/DATE: THURSDAY 18/04/2019 **2.30 PM – 5.30 PM**

INSTRUCTIONS:

- **Answer All Questions**
- **Adhere to the instructions on the answer booklet**

QUESTION ONE

(a) Find the range and domain for the function $f(x)=\sqrt{x^2-4x-32}$ [2 marks]

(b) Evaluate the following limits

(i) $l \Im_{x \to 1} \frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$ [2 marks]

(ii) $\lim_{x \to \infty} \frac{7x^6+x^6}{1-x^3}$ [2 marks]

(iii) $\lim_{x \to 0} \frac{\tan x}{4x}$ [2 marks]

(c) Given that $f(x)=\frac{1}{x^2+1}$, evaluate $\frac{dy}{dx}$ from first principles [3 marks]

(d) Find the gradient of the curve $y = x^x$ at the point. X=1 [2 marks]

(e) Given that $x = \sin^2\theta$ and $y = \cos^3\theta$, evaluate $\frac{dy}{dx} + \frac{3}{2} y^{\frac{1}{3}}$ [2 marks]

(f) Given the functions $f(x) = 2x^2 + y^2 = 6 \wedge y^2 = 4x$, show that the graphs of the two functions intersect orthogonally. [3 marks]

(g) A spherical balloon is being blown up so that its volume increases at the rate of $0.5\, cm^3/s$. Find the rate at which the radius increases. $V = \frac{4}{3}\pi r^3$ [2 marks]

QUESTION TWO

(a) Evaluate the following integrals

(i) $\int x^2 \sqrt{x^3 + 5}\, dx$ (use substitution) [2 marks]

(ii) $\int e^x \cos x\, dx$ (use by parts) [3 marks]

(iii) $\int \frac{x^3 - 5x^2 - x - 5}{x^2 - 1} dx$ [3 marks]

(iv) $\int \frac{dx}{x^2 - 3x + 2}$ [3 marks]

(v) $\int_0^2 \int_{-1}^1 (1 - 6x^2 y)\, dy\, dx$ [3 marks]

(b) Determine the area of the surface generated by revolving the curve $y = 3x ¿ x = 1\, to\, x = 4$ about the y-axis [3 marks]

(c) Evaluate the length of a circle defined by $x = a\cos t\,,\ y = a\sin t \quad 0 \le t \le 2\pi$

[3 marks]

QUESTION THREE

(a) Determine the angle between the vectors $\tilde{a} = i - 2j + 4k \wedge \tilde{b} = -4i + j - 2k$

(b) Find the value of t, for which the vectors $\tilde{a} = 2ti + 4j + 2k \wedge \tilde{b} = i + 3k - j$ are orthogonal hence find a unit vector orthogonal to the vectors $\tilde{a} \wedge \tilde{b}$

(c) Determine the volume of the parrallelopipe spanned by the vectors $i + j\,,\ j + k$ and $k + i$

(d) Solve the system of equations below by the cofactor expansion method.

(i) $x + 3y - 2Z = 1$

$4x - 5y + 6Z = 12$

$3x + 5y + 2z = 19$

(ii) Verify the solutions obtained in 3d(i) by crammers rule.

QUESTION FOUR

(a) State the order, degree and linearity of the differential equations below

(i) $Uxx + 4U_x = 0$

(ii) $1 + \left(\frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$

(b) Solve for y given that $\frac{dy}{dx} = x\cos x$

(c) Solve the differential equation $y' - 2xy = x$ using a suitable integrating factor.

(d) Determine whether the differential equation

$\left(5x^4+3x^2y^2-2xy^3\right)dx+\left(2x^3y-3x^2y^2-5y^4\right)dy=0$ is exact, hence solve,

(e) Solve the differential equations below

(i) $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=0$

(ii) $y''+4y'+5y=0$

(iii) $y''-3y'+2y=e^{3x}$
