CHUKA   UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF ARTS IN ECONOMICS AND SOCIOLOTY

ECON 234: MATHEMATICS FOR ECONOMISTS II

STREAMS: BA (ECON SOCI) Y3S1   **TIME: 2 HOURS**

DAY/DATE: FRIDAY 14/12/2018   **8.30 AM – 10.30 AM**

INSTRUCTIONS:

Answer Question One and any other Two

QUESTION ONE

(a)   Evaluate the following functions:

   (i) $Log_3 81 + Log_5 5$ [2 marks]

   (ii) $log_y \left(\frac{1}{y^3}\right)$ [1 mark]

(b)   Simplify the following functions

   (i) $\frac{x^8}{y^3}$ [2 marks]

   (ii) $\left(\frac{x^{-1/3}}{x^{-2/3}}\right)^6$ [2 marks]

(c)   Given the following consumption function, find the marginal propensity to consume and marginal propensity to save.

$C = 200 + 0.9y^d$

$T = 20 + 0.2y$ [4 marks]

(d)   Consider the following production function:

$Q = AL^{\propto}$ where $0 < \propto < 1;$ $A > 0$

(i)   Find the marginal product of labour (MPL)   [2 marks]

(ii)   Express MPL in terms of $\propto$ L and Q [2 marks]

(iii)   Determine the slope of MPL function   [2 marks]

(iv)   What is the sign of the slope of MPL?   [1 mark]

(v)   Determine whether MPL increases, diminishes or remains constant as L increases.
                                                                     [2 marks]

(e)   A consumer utility function is given by $U = q_1 q_2$

        Where q_1 and q2are the quantities of the two commodities consumed. If the price of q_1 is
        6 and that of q_2 is 3 and the budget is 60

        (i)   Write out a constrained utility maximization problem out of information given.
                                                                     [2 marks]

        (ii)   What is the corresponding augmented objective function?   [1 mark]

        (iii)   Find the levels of q_1 and q_2 that will satisfy the first-order condition for
                maximum.   [3 marks]

        (iv)   Is the second-order condition satisfied? Justify   [2 marks]

(v)   Compute the optimum value of U   [2 marks]

QUESTION TWO

(a)   Given the following information:

        P=12.1

        Total variance cost (TVC) $= -1/_{20} \, Q^3 - 1.5Q^2 + 17.5Q$
        Total fixed cost (TFC) $= 50$

        Find:

        (i)   The total cost (TC), total revenue and profit function.   [4 marks]

        (ii)   Find the output level at which profits are maximized   [3 marks]

(iii)   Compose the resulting MC (Marginal cost) and MR (Marginal Revenue) at the point of profit maximizing level of output.   [3 marks]

(b)   (i)   The Marginal Propensity to consume (MPC) for some economy is given as:
MPC=0.75

Determine the consumption function for the economy given that C=100 when y=0
[4 marks]

(ii)   Determine the consumers surplus for the demand function Q+P=8
Where P = 3   [6 marks]

QUESTION THREE

(a)   Given the following production function $Q = 40K^{1/2} L^{1/2}$

(i)   Find the marginal product of capital   [2 marks]

(ii)   Find the marginal product of labour   [2 marks]

(iii)   Determine the marginal Rate of Technical substitutions.   [3 marks]

(iv)   Set Q=80 and find the corresponding Isoquant.   [2 marks]

(v)   Find the marginal rate of Technical substitution for $L = 1$ [1 mark]

(vi)   Does the isoquant obey the law of diminishing MRTS?   [1 mark]

(b)   Given the following functions:

$$y = 3x^3 - 36x^2 + 135x - 13$$
$$y = x^3 - 3x + 4$$

(i)   Find the critical values of x for each function   [5 marks]

(ii)   Establish whether such critical values gives rise to relative maximum or minimum.   [5 marks]

QUESTION FOUR

(a)   The demand and total cost function for a firm are given by:

$$P = 7 - \frac{2}{5}Q$$

$$TC = \frac{4}{7}Q^3 - \frac{3}{4}Q^2 + 7Q + 5$$

Find:
(i)   The level of Q and P that will maximize profits.   [2 marks]
(ii)   The level of Q that will maximize total revenue.   [2 marks]
(iii)   The level of Q that will minimize AVQ   [2 marks]
(iv)   The level of Q that will minimize MC   [2 marks]
(v)   The minimum AVC and MC   [2 marks]

(b)   (i)   The value of cheese that improves with age is given by:

$$V = 1400 \, (1.25)^{\sqrt{t}}$$

If the cost of capital under continuous compounding is 9% a year and there is no storage costs for aging of the cheese in the company, how long should the company store the cheese?

The company has to maximize the value of cheese which is given by $P = Ve^{-rt}$
                                                                                                       [7 marks]

        (ii)   Given the following function: $Q = 52 - 0.2Y$ at $y = 10$. Determine the income elasticity of demand and interpret your results.   [3 marks]