CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS 2023.

FIRST YEAR EXAMINATIONS FOR THE AWARD OF BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING.

MATH 407: FOURIER ANALYSIS **TIME: 2 HOURS**

INSTRUCTIONS

Answer question one and any other two questions

Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

a. Obtain a_0, a_n and b_n for the Fourier series of the function defined as 6mks

$$f(x)=\begin{cases}0, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

b. Find Fourier Sine transform of $f(x)=2e^{-3x}+3e^{-2x}$ 5mks

c. Find Fourier cosine transform of $f(x)=\begin{cases}1, & 0<x<a\\ 0, & x>a\end{cases}$ 4mks

d. Find $f(x)$ if its finite Fourier sine transform is given by $F_s(p)=\dfrac{1-\cos p\pi}{p^2\pi^2}$ *for* $p=1,2,3.....$ and $0<x<\pi$

2mks

e. If $F(s)$ is the complex Fourier transform of $f(x)$, show that $F[f(ax)=\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$ 5mks

f. Using Parseval's identity for sine transforms, obtain $f(x)=\displaystyle\int_0^\infty \frac{x}{\left(x^2+1\right)^2}$ given that $f(x)=\displaystyle\int_0^\infty \frac{x}{\left(x^2+1\right)^2}$

and $F_s(s)=\dfrac{\pi}{2}e^{-s}$ 5mks

g. Determine the exponential form of the Fourier series for the function defined by $f(t)=e^{2t}$ when $-1<t<1$ and has period 2 5mks

QUESTION TWO

a. A periodic function of period 4 is defined as $f(x)=\begin{cases}x, & 0\le x\le 2\\ -x, & -2\le x\le 0\end{cases}$,

Obtain a_0, a_n and b_n 6mks

b. The temperature $u(x,t)$ in a semi-infinite rod $0<x<\infty$ is determined by the differential equation $u_t(x,t)=2u_{xx}$ subject to conditions:

$u=0,\ when\ t=0,\ x\geq 0$

$u_t=-k,\ when\ x=0,\ t>0$

Obtain the equation for the temperature $u(x,t)$ at any point along the rod 10mks

c. Find the function f(x) if its Fourier sine transform is given by e^{-as} 4mks

QUESTION THREE

a. Using Fourier transform, solve the equation $u_t(x,t)=ku_{xx},\ 0<x<\infty,\ t>0$ subject to the conditions

$u(0,t)=0,\ t>0$,

$u(x,0)=e^{-x},\ x>0$,

$u\ and\ u_x$ tends to zero as $x\to\pm\infty$ 10mks

b. Solve $u_t=u_{xx},\ 0<x<6,\ t>0$ under the given conditions $u_x(0,t)=0,\ u_x(6,t)=0,\ u(x,0)=2x$ by Fourier transforms. 9mks

QUESTION FOUR

a. A periodic function f(t) of period 2 is defined by $f(t)=\begin{cases}3t, & 0<t<1\\ 3, & 1<t<2\end{cases}$, Obtain a_0, a_n and b_n 6mks

b. Find the Fourier series expansion of the periodic function of period 2π given as

$f(x)=x^2,\ -\pi\leq x\leq\pi$

Hence, find the sum of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}\ldots$ 8mks

c. Find the Fourier sine integral for $f(x)=e^{-\beta x},\quad \beta>0$ 6mks

QUESTION FIVE

a. Solve $U_t=kU_{xx}\quad for\ x\geq 0,\ t\geq 0$, under the given conditions $U=U_0\ at\ x=0, t>0$, with initial conditions $U(x,0)=0, x\geq 0$ by Fourier transforms. 8mks

b. Find the finite Fourier sine transform of f(x) = 1 in (0, π). Use the inversion theorem and find the Fourier series for f(x) = 1 in (0, π). Hence show that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....=\frac{\pi}{4}$ 6mks

c. Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate the Fourier sine transform of $xe^{-a^2x^2}$ 6mks