CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 205: ELEMENTS OF SET THEORY

STREAMS: "AS ABOVE" **TIME: 2 HOURS**

DAY/DATE: TUESDAY 29/08/2023 **11.30 A.M – 1.30 P.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) Differentiate the following terms and give examples
   i. Linearly ordered set and a partially ordered set
   ii. Countable and uncountable set (6 marks)

b) Given the sets $A_n = \{n+1, n+2, n+3 \ldots\}$, where n is a positive integer evaluate
   i. $\bigcup_{n=3}^{10} A_n$ and $\bigcap_{n=1}^{10} A_n$
   ii. $\bigcup_n A_n$ and $\bigcap_n A_n$ (6 marks)

c) Find the domain of the function $f : R \to R$ defined by $f(x) = \sqrt{9 - x^2}$ (2 marks)

d) Consider the set $A = \left\{4 + (-1)^n \frac{n-1}{n}\right\}$ where n is a positive integer
   i. Find the supremum and the infimum of A (4 marks)
   ii. Find all the limit points of A (2 marks)

e) With an appropriate example, show that a bounded sequence is not necessarily convergent (2 marks)

f) Prove that the set of integers is countable (3marks)

g) Prove that if the limit of a sequence exists, then it is unique (3marks)

h) State the Axiom of choice (2 marks)

QUESTION TWO (20 MARKS)

a) Let n be a positive integer. Suppose a function L is defined recursively as

$$L(a,b) = \begin{cases} 0 & if\ a < b \\ L(a-b,b) & a \geq b \end{cases}$$

Find

i. L(2,3) (1 mark)

ii. L(14,3) (2 marks)

iii. What does L do? (2 marks

iv. Find L(5861,7) (2marks

b) Prove the De'Morgans law $(\bigcup_k A_k)^c = \bigcap_k A_k^c$ of sets (5 marks)

c) Let $A_m = \{x\epsilon\mathbb{R}: 0 \leq x \leq 1 - \frac{1}{k}\}$, determine and explain the following sets as intervals

i. $A_3 \cap A_7$ (2 marks)

ii. $A_3 \cup A_7$ (2 marks)

iii. $\bigcup_m A_m$ (2 marks)

iv. $\bigcap_m A_m$ (2 marks)

QUESTION THREE (20 MARKS)

a) Prerequisites in a college are partial ordering of available classes. Denote A is a requisite of B by $A \prec B$. Let C be the ordered set of mathematics courses and their prerequisite as shown below.

Class	Math 101	Math 201	Math 250	Math 251	Math 340	Math 341	Math 450	Math 500
Prerequisite	None	Math 101	Math 101	Math 250	Math 201	Math 340	Math 201,250	Math 250,251

Required:

i. Draw an Hasse diagram for the partial ordering of these classes (4 marks)

ii. Find all the minimal and maximal elements of these classes (3 marks)

iii. Determine the first and last element if they exist. (2 marks)

b) Let A and B be sets. Show that the product order on $A\times B$ defined by $(a,b)\prec(c,d)$ if $a\leq c$ and $b\leq d$ is a partial order on $A\times B$ (6 marks)

c) Let λ be an ordinal number. Prove that $\lambda+1$ is the immediate successor of λ (5 marks)

QUESTION FOUR (20 MARKS)

a) Given the functions $r: x\rightarrow \frac{1}{2}x-4$ and $r\circ s: x\rightarrow \frac{9x-1}{2x}$, find $s(x)$. (4 marks)

b) Find if the following sets are bounded or not and if bounded find the $sups$ and $infs$

(i) $S_1=\{x\in\mathbb{R}: a\leq x<b\}$ (2mks)

(ii) $S_2=\left\{1+(-1)^n\frac{1}{n}: n\in\mathbf{N}:\right\}$ (2mks)

(iii) $S_2=\{1+(-1)^n.n: n\in\mathbf{N}:\}$ (2mks)

c) Define a countable set. Hence illustrate that the set of even integers is countable 6 marks)

d) Prove that the intervals [0,1]and(0,1] are equivalent. (4 marks)
