CHUKA

UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (SCIENCE AND ARTS)

MATH 424: NUMERICAL ANALYSIS II

STREAMS: BED (SCI & ARTS) AB1, AB5   **TIME: 2 HOURS**

DAY/DATE: MONDAY 17/12/2018   **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS: Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE (COMPULSORY) – 30 MARKS

(a)   Use the method of least squares to find the equation that fits the data in the table

[5 marks]

x	0	1	2	3	4
y	1	5	10	22	38

(b)   Compute the Tchebysher polynomials $T_3(x)$ given that $T_n(x)=2xT_{n-1}(x)-T_{n-2}(x)$

and $T_o(x)=1$ and $T_1(x)=x$ [6 marks]

(c)   Determine the best lower approximation $f(x)=2x^3+x^2$ on the interval $[-1,1]$

[4 marks]

(d)   Consider the data in the table and use Lagrange's interpolation to find x given that

$y=2.75$ [6 marks]

	0.0	1.0	1.5	2.0

x				
y	2.5	3.8	4.6	5.9

(e)   Compute the eigenvalues and eigenvectors of the matrix   [5 marks]

$$A = \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}$$

(f)   Express x^4 in a series of $T_n(x)$ using the recurrence Tchebysher formula given that

$$x T_o(x) = T_1(x)$$ [4 marks]

QUESTION TWO (20 MARKS)

(a)   Given a general system of linear equation $A x = b$

      (i)   Outline the LU factorization method   [2 marks]

      (ii)   Solve the system of equations using the Crout's method   [6 marks]

$$4 x_1 + 2 x_2 + 6 x_3 = 16$$

$$2 x_1 + 82 x_2 + 39 x_3 = 206$$

$$6 x_1 + 39 x_2 + 26 x_3 = 113$$

(b)   Consider the data in the table below

x_i	$f(x_i)$	$f'(x_i)$
8.3	17.565	3.116
8.6	18.505	3.156

      (i)   Construct the Hermite polynomial that fits the data based on divided differences

              [4 marks]

      (ii)   Use the polynomial obtained in b(i) above to approximate $f(8.4)$ [2 marks]

(c)   Use the principle of least square to fit a parabola of the form ax^2+bx^2c for the data

[6 marks]

x	1.0	1.6	2.5	4.0	6.0
y	9.4	11.8	14.7	18.0	23.0

QUESTION THREE (20 MARKS)

(a)   Use the Cholesky method to solve system   [7 marks]

$$x_1+3x_2+4x_3=15$$

$$2x_1-107x_2+3x_3=37$$

$$3x_1+2x_2-10x_3=-10$$

(b)   Approximate x for $f(x)=2800$ using the method of successive approximation of

the   data in the table below   [8 marks]

x	10	15	20
y	1754	2654	3564

(c)   Obtain the rational approximation of the farm $\dfrac{a_2x^2+a_1x+a_o}{b_1x+1}$ for the function

$f(x)=x^2$ to e^x

[5 marks]

QUESTION FOUR (20 MARKS)

(a)   (i)   Define a diagonally dominant matrix and state its significance   [2 marks]

(ii)   Determine whether or not $\begin{bmatrix} 5 & -2 & 3 \\ -3 & 9 & 1 \\ 2 & -1 & -7 \end{bmatrix}$ is diagonally dominant   [2

marks]

(iii)   Hence……….. the system of equation using the Gauss Jacobi method with

$x^{(o)}=[0,0,0]^T$

$$5x-2y+3z=-1$$

$$-3x+9y+z=2$$

$$2x-y+7z=3$$ [8 marks]

(b) Find the largest eigenvalue and the corresponding eigenvector of the matrix using power

method with $[1,1,1]$ $A=\begin{pmatrix} 2 & 2 & 1 \\ 4 & 3 & 4 \\ 7 & 6 & 1 \end{pmatrix}$ [8

marks]

QUESTION FIVE (20 MARKS)

(a) Solve the system of linear equations using the Gauss Siedel method [9 marks]

$$14x-3y+5z=4$$

$$3x-10y-2z=12$$

$$5x+2y+20z=16$$

(b) Use the Lagranges interpolation method to find x given that $y=2.75$ [6

marks]

x	0.0	1.0	1.5	2.0
y	2.5	3.8	4.6	5.9

(c) Obtain the least linear approximation to the function $f(x)=\sin \pi x$ on $[0,1]$ [5

marks]