

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

**FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF
BACHELOR OF EDUCATION (SCIENCE AND ARTS), BACHELOR OF SCIENCE
GENERAL**

MATH 420: PARTIAL DIFFERENTIAL EQUATIONS I**STREAMS: BED (SCI, ARTS), BSC (GEN)****TIME: 2 HOURS****DAY/DATE: FRIDAY 14/12/2018****2.30 PM – 5.30 PM****INSTRUCTIONS:****ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS****QUESTION ONE (COMPULSORY) (20 MARKS)**

- (a) Use the method of separation of variables to solve the Pfaffian equation
 $yz dx + 2zx dy - 2xy dz = 0$ [5 marks]
- (b) Find the partial differential equation associated with the following primitives
- (i) $z = ax^3 + by^2$ [4 marks]
- (ii) $\phi\left(z^2 - xy, \frac{x}{z}\right) = 0$ [5 marks]
- (c) Find the second partial derivatives of $z = x^2 y - y^{\frac{7}{n}} x^2 + 43x - 7 \tan xy$ [4 marks]
- (d) Verify that the equation is integrable $(y^2 + xz) dx + (x^2 + yz) dy + 3x^2 dz = 0$ [4 marks]
- (e) Determine whether or not the functions $V = x^2 + y^2 - z^2$ is a solution to the PDE [3 marks]

$$(x-2z)\frac{dz}{dx}-(x+z)\frac{dz}{dy}=x-y$$

- (f) Find the solution to the partial differential equation [5 marks]

$$y^2 z p - x^2 z q = x^2 y$$

QUESTION TWO (20 MARKS)

- (a) Verify that the equation is integrable and determine its solution. [8 marks]

$$(2xyz+z^2)dx+x^2zdy+(xz+1)dz=0$$

- (b) Find the complete solution of the PDE $p^2 q^2 + q^2 = 1$ [7 marks]

- (c) Find the PDE equation associated with the primitive [5 marks]

$$z=f(2x+y)+g(3x-y)$$

QUESTION THREE (20 MARKS)

- (a) Find the integral surface of the equation $(y+z)p+(x+x)q=x+y$. Passing through $x+y=0 \wedge z=1$ using the multipliers $(1,-1,0)$ and $(1,0,-1)$ [9 marks]

- (b) (i) Define the compatibility of a system of first order partial differential equations. [2 marks]

- (ii) State the condition for compatibility of two PDE. [2 marks]

- (iii) Show that $xp=yq$ and $z(xp+yq)=2xy$ are compatible. [4 marks]

- (c) Find the complete integrals of the PDE $z - px - qy = \sqrt{1+p^2+q^2}$ [3 marks]

QUESTION FOUR (20 MARKS)

- (a) Use the Lagrange method to solve

$$\frac{y-z}{yz}p + \frac{z-x}{xz}q = \frac{x-y}{xy} \quad [8 \text{ marks}]$$

- (b) Find the surface which intersects the surface of the system $z(x+y) = c(3z+1)$ orthogonally and passes through the curves $x^2+y^2=1$ and $z=1$ [4 marks]

- (c) Solve the partial differential equation $p^2+q^2=x+y$ [8 marks]

QUESTION FIVE (20 MARKS)

- (a) Use the Charpit's method to find a complete integral of the equation $xp^2+yq^2=z$ [9 marks]

- (b) Identify the special PDE and find the complete solution $\sqrt{p}+\sqrt{q}=1$ [8 marks]

- (c) Show that $v=f(y-3x)$ where f is an arbitrary function is the general solution to the equation $\frac{\partial v}{\partial x} + \frac{3\partial v}{\partial y} = 0$ [3 marks]
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