

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN
ACTUARIAL SCIENCE

ACMT 302: ACTUARIAL MATHEMATICS II

STREAMS:

TIME:2 HOURS

DAY/DATE: THURSDAY 20/04/2023

8.30 A.M. –10.30 A.M.

INSTRUCTIONS

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

(a) Discuss the differences between prospective and retrospective reserve. (4 Marks)

(b) Consider a Markov process with state space and transition matrix, P :

$$P = \begin{pmatrix} p & q & 0 \\ 0.5 & 0 & 0.5 \\ p-0.5 & 0.7 & 0.2 \end{pmatrix}$$

(i) Determine the values of p and q . (2 Marks)

(ii) Calculate the transition probabilities. (3 Marks)

(iii) Draw the transition graph for the process represented by P . (2 Marks)

(c) Calculate the annual premium for a term assurance with a term of 10 years to a male aged 30, with a sum assured of £500,000, assuming AM92 Ultimate mortality and interest of 4% pa. Assume that the death benefit is paid at the end of the year of death. (6 Marks)

(d) Consider the extract from a double decrement table given below, where d and w refer to death and withdrawal respectively.

Age, x	$(al)_x$	$(aq)_x^d$	$(aq)_x^w$
55	100,000	275	3,100
56	96,625	380	2,870
57	93,375		

Calculate:

i. $(ap)_{56}$ (2 Mark)

ii. $2(aq)_{55}^w$ (2 Mark)

(e) A life insurance company issues a four-year unit-linked policy to a life aged 51 exact. The policy has the following profit vector:

$$(1.798.01, -401.56, -355.10, -1.075.23)$$

Determine the net present value of the profits of this policy, assuming that the company sets up reserves in order to zeroise future negative expected cash flows on the policy. (6 Marks)

Basis:

Mortality	AM92 Ultimate
Rate of interest on non-unit fund cash flows	2.5% per annum
Risk discount rate	4.5% per annum

(f) A life insurance company uses the three-state healthy-sick-dead model described above to calculate premiums for a 3-year sickness policy issued to healthy policyholders aged 60. Let S_t denote the state occupied by the policyholder at age $60 + t$, so that $S_0 = H$ and $S_t = H, S$ or D for $t = 1, 2, 3$. The transition probabilities used by the insurer are defined in the following way:

$$p^{jk}_{60+t} = P(S_{t+1} = k | S_t = j)$$

For $t = 0, 1, 2$, it is assumed that:

$$p^{HH}_{60+t} = 0.9 \quad p^{HS}_{60+t} = 0.08 \quad p^{SH}_{60+t} = 0.7 \quad p^{SS}_{60+T} = 0.25$$

Calculate the probability that a new policyholder is sick at exact age 62. (3 Marks)

QUESTION TWO (20 MARKS)

(a) A life aged exactly 50 buys a 15-year endowment assurance policy with a sum assured of £50,000 payable on maturity or at the end of the year of earlier death. Level premiums are payable monthly in advance. Calculate the monthly premium assuming AM92 Ultimate mortality and 4% pa interest. Ignore expenses. (6 Marks)

(b) A life insurance company issues 20-year temporary assurance policies to lives aged 45. The sum assured, which is payable immediately on death, is £400,000 for the first 10 years, and

£100,000 thereafter. Level annual premiums are payable in advance for 20 years, or until earlier death. The premium basis is:

Mortality: AM92 Ultimate Interest: 4% per annum Expenses: None

- i. Show that the premium payable is approximately £870.25 per annum. (6 Marks)
- ii. Find the net premium reserve ten years after the commencement of the policy, immediately before the payment of the eleventh premium, assuming the reserving basis is the same as the premium basis. (5 Marks)
- iii. Give an explanation of your numerical answer to part (ii). Describe the disadvantages to the insurance company of issuing this policy. (3 Mark)

QUESTION THREE (20 MARKS)

- (a) In a certain population, forces of decrement are assumed to be constant over individual years of age. The following independent forces of decrement will be assumed for this population between the exact ages of 60 and 62:

Force of decrement for year of age commencing from exact age x

Age, x	Due to mortality	Due to sickness
60	0.011	0.065
61	0.013	0.071

Construct a double decrement table including the two decrements of mortality and sickness, for this population between exact ages 60 and 62, assuming a radix of $(al)_{60}=100,000$ (7 Marks)

- (b) We wish to extend the multiple decrement table constructed in the previous question (incorporating the decrements of death **(d)** and sickness **(s)**) to include the decrement of withdrawal **(w)**. It is believed that the independent forces of withdrawal will conform to those underlying the withdrawal decrement in the multiple decrement table below:

Age x	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
60	100,000	175	2,490
61	97,335	180	2,160
62	94,995		

Calculate the first two lines of the triple decrement table (i.e., between ages 60 and 62) incorporating the three decrements **d**, **s** and **w**, assuming that the forces of sickness and mortality are unchanged. (13 Marks)

QUESTION FOUR (20 MARKS)

A life insurance company sells five-year-term, single-premium, unit-linked policies each for a premium of £10,000. There is no bid/offer spread and the allocation percentage is 100%. The only charge is a 2% annual management charge. The maturity, death and surrender benefits are equal to the value of the units at maturity, or at the end of the year of death or surrender, as appropriate, after deduction of the annual management charge in each case.

- i. Assuming unit growth of 9% pa, calculate the value of the units at the start and end of each year after deduction of the management charge, and the amount of management charge each year. (8 Marks)
- ii. Calculate the net present value of the contract assuming:
 - Commission of 5% of the premium
 - Initial expenses of £50
 - Annual renewal expenses of £20 in the 1st year, inflating at 5% pa
 - Independent probability of mortality is 0.5% at each age
 - Independent probability of surrender is 5% at each age
 - Non-unit fund interest rate is 9% pa
 - Risk discount rate 12% pa

The company holds unit reserves equal to the full value of the units (after deduction of annual management charge) and zero non-unit reserves. You may assume that expenses are incurred at the start of the year and that death and surrender payments are made at the end of the year.

(12 Marks)

QUESTION FIVE (20 MARKS)

A life insurance company issues a with profit whole life assurance policy to a life aged 50 exact. The sum assured is \$ 75,000 together with any attaching bonuses and is payable immediately on death. Level premiums are payable monthly in advance ceasing on the policyholder's death or on reaching age 80 if earlier.

Simple annual bonuses are added at the end of each policy year (i.e., the death benefit does not include any bonus relating to the policy year of death).

The company calculates the premium on the following basis:

Mortality	AM92 Select
Interest	4% per annum
Expenses	
Initial	\$270
Renewal	\$65 at the start of the second and subsequent policy years and payable until death
Claim cost	\$205 on death
Commission	
Initial	75% of the total premium payable in the first policy year
Renewal	2.5% of the second and subsequent monthly premiums
Bonuses	Simple bonus of 2.0% of basic sum assured per annum

- i. Calculate the monthly premium for this policy. (13 Marks)
- ii. Calculate the gross prospective policy value at the end of the 30th policy year given that the total actual past bonus additions to the policy have followed the assumptions stated in the premium basis above (including the bonus just vested). (7 Marks)

Policy value basis:

Mortality	AM92 Ultimate
Interest	4% per annum
Expenses	
Initial	\$275
Renewal	\$75 at the start of the second and subsequent policy years and payable until death
Claim cost	\$255 on death
Commission	
Renewal	2.5% of the second and subsequent monthly premiums
Bonuses	Simple bonus of 2.5% of basic sum assured per annum