CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE/ ARTS, BACHELOR OF SCIENCE

MATH 412: DIFFERENTIAL GEOMETRY

STREAMS: BED (ARTS, SCI) BSC **TIME: 2 HOURS**

DAY/DATE: MONDAY 10/12/2018 **11.30 AM – 1.30 PM**

INSTRUCTIONS:

- **Answer Question One and any other Two Questions**
- **Do not write on the question paper**

Question One (30 Marks)

a. Define a regular representation function on an interval I . Hence show that the function $x=(t+1)e_1+\text{¿}\overset{t}{(\text{¿¿}2+3)}e_2$, $-\infty\leq t\leq\infty$ is a regular parametric representation.

(3 marks)

b. When is a real valued function $t=t(\theta)$ on an interval I_θ said to have an allowable change of parameter? Take $t=(b-a)\theta+a,\quad 0\leq\theta\leq1$, $a<b$ to illustrate this.

(5 Marks)

c. Given the space curve $x=(a\cos t)e_1+(a\sin t)e_2+bt\,e_3$. Find its arc length for $0\leq t\leq2\pi$.

(3 marks)

d. State without proof the Fundamental existence and uniqueness Theorem

(2marks)

e. Prove that if $x=x(s)$ is a natural representation on an interval I_s , then $¿s_2-s_1\vee¿$ is the length of the arc $x=x(s)$ between points corresponding to $x(s_1)\wedge x(s_2)$ (3 marks)

f. Find the curvature and torsion of the curve $x=(3t-t^3)e_1+3t^2e_2+(3t+t^3)e_3$ and comment about your results.
(6 Marks)

g. Show that along a regular curve $x=x(s)$,
$\dddot{x}=-k^2t+\acute{k}n+\tau kb$ (5 Marks)

h. Show that the First Fundamental form is positive definite. (3 marks)

QUESTION TWO (20 Marks)

a. Find the equation of the oscillating plane of the helix $x=(cost)e_1+(sint)e_2+te_3$ at $t=\frac{\pi^c}{2}$
(6 marks)

b. Derive the First Fundamental form I to the coordinate patch $x=x(u,v)$ on a surface of class ≥ 2 .
(7 marks)

c. Consider the surface represented by $x=ue_1+ve_2+(u^2-v^2)e_3$. Find its second fundamental form II
(7 marks)

QUESTION THREE (20 Marks)

a. Prove that the First Fundamental form depends only on the surface and not on the particular representation
(10 marks)

b. Consider the helix $x(t)=acost\,e_1+asint\,e_2+te_3$ **.** Find the equation of the principal normal and the osculating plane at $t=\frac{\pi}{2}$
(10 marks)

QUESTION FOUR (20 Marks)

(a) Define torsion $\tau(s)$ of the curve C at the point **x**(s). Hence show that the sign of τ is independent of the sense of principal vector **n** and the orientation of C. (6 marks)

(b) Find the equations of the tangent line and normal plane to the curve $x = t e_1 + t^2 e_2 + t^3 e_3$ at $t = 1$ (6 marks)

(c) Define an involute of a curve C and show that its curvature [of an involute $x^{¿} = x + (c-s)t$ of $x = x(s)$] is given by

$$k^{¿^2} = \frac{k^2 + \tau}{(c-s)^2 k^2}$$

(8 marks)

QUESTION FIVE (20 Marks)

a. (i) Define a curvature vector on the curve C at the point **x**(s). Hence show that the curvature vector is independent of orientation. (3 marks)

(ii) Show that the curvature of the curve $x = a(cost)e_1 + a(sint)e_2, a > 0$ is $\frac{1}{a}$

(5 marks)

b. Show that along a regular curve

$$¿ x' \times x'' \vee ¿^2$$

$$¿ x' \times x'' \times x''' \vee \frac{¿}{¿}$$

$$x = x(s), the\, torsion\, \tau = ¿$$

(12 marks)
