CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS AND STATISTICS, BACHELOR OF EDUCATION ARTS & SCIENCE & BACHELOR OF ARTS

MATH 344: THEORY OF ESTIMATION

STREAMS: BSC (ECON & STATS), BED (ARTS& SCI) BA Y3S1 TIME: 2 HOURS

DAY/DATE: FIRDAY 07/12/2018 8.30 A.M. – 10.30 A.M.

INSTRUCTIONS: Attempt question ONE and any other TWO

QUESTION 1 (30 MARKS)

(a) Briefly explain the following [4 marks]

(i) T is an estimator of parameter θ

(ii) T is an unbiased estimator of parameter θ

(b) Suppose that one can define the mean squared error MSE $(X)=E\{(X-\theta)^2\}$ for any estimator of θ. Now, given that T is an unbiased estimator of θ and that $Var(T)=k\theta^2$. Determine an expression for $MSE(CT)$ and the value of C for which $MSE(CT)$ is minimum where C is some constant [5 marks]

(c) Let $X_1, X_2, \ldots, X_n$ be n independent observations of a random variable X which assumes two values 0 and 1 with respective probabilities q and p such that

$p+q=1$. Show that $\frac{T(T-1)}{n(N-1)}$ is an unbiased estimator of p^2 where $T=\sum_{i}^{n} x_i$.

[6 marks]

(d) Let T_1 and T_2 be two independent unbiased estimators of the same parameter θ such that $Var(T_1)=2Var(T_2)$. Find the values of the constants K_1 and K_2 such $T=K_1 \quad T_1+K_2T_z$ is unbiased estimator of θ with the minimum possible variance

[5 marks]

(e) Prove by contradiction that the minimum variance unbiased estimator (MVUE) is unique if it exists [5 marks]

(f) Let $X_1, X_2, \ldots, X_n$ be n independent observations of a random variable X from a normal distribution with mean μ and variance σ^2. Show that the sample mean $\acute{x}$ is a consistent estimation of the population mean μ provided the variance is finite.

[5 marks]

QUESTION 2 (20 MARKS)

(a) Let X be a Poisson variate with parameter $\lambda\, i.e\, X \sim P(\lambda)$

(i) Verify whether $T=\sum_i x_i$ is a sufficient statistic for λ . Here assume that T is poisson with parameter $(n\lambda)$ i.e $X \sim P(n\lambda)$.

[5 marks]

(ii) Show that $\acute{x}$ is the minimum variance bound unbiased estimator (MVBUE)

[5 marks]

(b) Consider a normal random variance X with mean μ and variance θ^2. Where θ^2 is known. Find the sufficient statistic for μ . (Hint: use the Pitman-Koopman form of distribution).

[10 marks]

QUESTION 3 (20 MARKS)

(a) State any 3 properties of the maximum likelihood estimates [3 marks]

(b) Suppose a random sample of size n is drawn from a probability distribution function given by

$$f(x)=\begin{cases}\dfrac{\theta^{2x}e^{-\theta^2}}{x!} & ,x=0,1,2\ldots\\ 0 & ,elsewhere\end{cases}$$

Find the maximum likelihood estimator for θ [10 marks]

(c) Consider a random variable X whose probability distribution function is given by

$$f(x)=\begin{cases}e^{-(x-\lambda)} & ,0\le x<\lambda\\ 0 & ,elsewhere\end{cases}$$

Find the maximum likelihood estimator for λ if a sample of size n is considered [7 marks]

QUESTION 4 (20 MARKS)

(a) Let $X_1, X_2, \ldots X_{n_1}$ be a random sample of size n_1 taken from a normal distribution with mean μ_1 and a known variance σ_1^2. On the other hand, let Y_1, $Y_2, \ldots Y_{n_2}$ be a random sample of size n_2 taken from a normal distribution with mean μ_2 and a known variance σ_2^2. Derive the $100(1-a)$ confidence interval for $(\mu_1-\mu_2)$. [10 marks]

(b) Let $\acute{x}$ the mean of a random sample of size n with parameters (μ,σ^2) from a normal population. Find the sample size n such that $P[(\acute{x}-1)<\mu<(\acute{x}+1)]=0.9$ [10 marks]

QUESTION 5 (20 MARKS)

(a) Briefly describe the method of moments for the estimation of parameters. [8 marks]

(b) Explain the main disadvantage of the method of moments for parameter estimators. [2 marks]

(c) Let $X \quad G(a,\lambda)$. Find the parameter estimates for a and λ using the method of moments

[10 marks]
