

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS), BACHELOR OF EDUCATION (ARTS) AND SCIENCE

## MATH 325: FLUID MECHANICS I

STREAMS: BSC (MATHS), BED (ARTS \& SCI)
TIME: 2 HOURS
DAY/DATE: MONDAY 17/12/2018
8.30 A.M. - 10.30 A.M.

## INSTRUCTIONS:

- Answer question ONE (Compulsory) and any other TWO questions
- Adhere to the instructions on the answer booklet


## QUESTION ONE (30 MARKS)

(a) Calculate the specific weight and specific mass of a liquid having a volume of $10 \mathrm{~m}^{3}$ and a weight of 50 KN .
[4 marks]
(b) A plate having an area of $0.4 \mathrm{~m}^{2}$ is sliding down the inclined plane a velocity of $0.5 \mathrm{~m} / \mathrm{s}$. there is a cushion of fluid 1.2 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the is 300 N
(c) The velocity distribution for flow over a plate is given by $u=\sqrt{2 y^{2}+y 3}$, where $u$
is the velocity in $\mathrm{m} / \mathrm{s}$ at a distance ${ }^{y}$ metres above the plate. Determine the velocity gradient and shear stress at the boundary and at 2 m from the plate. Take $\mu=0.6 \mathrm{Nn} / \mathrm{m}^{2}$
(d) A soap bubble 20 mm diameter has an internal pressure in excess of the outside pressure of $10 \mathrm{~N} / \mathrm{m}^{2}$. Find the tension in the soap film.
(e) A u tube is made up of two capillaries of bores 1.2 mm and 2.4 mm respectively. The tube is held vertical and partially filled with liquid of surface tension $0.06 \mathrm{~N} / \mathrm{M}$; and zero contact angle. If the estimated difference in the level of the two menisci is 15 mm . Determine the mass density of the liquid
(f) In a fluid the velocity field is given by

$$
v=(2 x+3 y) i+\left(3 z+2 x^{2}\right) i+(2 t-3 z) k
$$

Determine the speed at time $\mathrm{t}^{i 2 s \text { at the point }(0,0,3) \text { [3 marks] }}$
(g) Obtain the equation to the streamlines for the velocity field given as $v=2 x^{3} i-6 x^{2} y j$
(h) Given $u=i\left(y^{2}+z^{2}\right)$ and $w=i\left(x^{2}+y^{2}\right)$. Find the most general form of $V$ so that the flow is possible for a steady three dimensional incompressible flow

## QUESTION TWO

(a) Given that $u=4 x\left(x^{2}-3 y^{2}, v=4 y\left(3 x^{2}-y^{2}\right)\right.$, examine whether these velocity components represent a physically possible two dimensional flow and whether the flow is rotational or irrotational [5 marks]
(b) Given that $u=x y, v=2 y z, \quad$ examine whether these velocity components represent two or three dimensional incompressible flow. If three dimensional determine the third component.
(c) The velocity components for a fluid flow are $u=1+2 y-3 z, v=4-2 x-5 z$,

$$
w=6+3 x-5 y
$$

(i) Show that it is possible case of fluid flow [2 marks]
(ii) Determine whether the fluid flow is irrotational and find the vorticity and rotation
marks]
(d) A two dimensional incompressible flow is given by $V_{r}=2 r \sin \theta \cos \theta$,

$$
V_{\theta}=-2 r \sin ^{2} \theta
$$

Determine whether these velocity components represents a physically possible flow field marks]

## QUESTION THREE

(a) A stream function $\Psi=4 x y$ locate the point at which the velocity vector has a magnitude of 7 units and makes an angle of $150^{\circ}$ with $x$-axis marks]
(b) The velocity potential function for a two dimensional flow is $\phi=x(2 y-1)$. At point

$$
p(4,5) \quad \text { determine }
$$

(i) The velocity [2 marks]
(ii) The value of the stream function [4 marks]
(c) The velocity function for a two dimensional flow is given by $\phi=x^{2}-y^{2}$
(i) Determine the velocity components in $x$ and $y$ directions marks]
(ii) Show that the velocity components satisfy the conditions for flow of continuity and irrotationality
marks]
(iii) Determine the stream function

## QUESTION FOUR

(a) Water is flowing in a pipe of 100 mm diameter with a mean velocity of $4 \mathrm{~m} / \mathrm{s}$ and at a gauge pressure of $300 \mathrm{kN} / \mathrm{m}^{2}$. Determine the total head if the pipe is 10 metres above the datum line. Neglect friction
(b) A pipeline shown below is 15 cm in diameter and it is at an elevation of 100 m at section A. At section $B$ it is at an elevation of 107 m and has a diameter of 30 cm . when a discharge of 50 litres $/ \mathrm{sec}$ of water is passed through this pipeline, pressure at A is 35 Kpa . The energy loss in the pipe is 2 m of water. Calculate the pressure at B if the flow is from A to B
(c) A pipe 200 m long slopes down at one in hundred and tapers from 600 mm diameter at the higher end to 300 mm diameter at the lower end and carries 100 litres $/ \mathrm{sec}$ of oil ( sp . Gravity 0.8 ). If the pressure gauge at the higher end reads $60 \mathrm{KN} / \mathrm{m}^{2}$. Determine
(i) Velocity at the two ends
(ii) Pressure at the lower end

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## QUESTION FIVE

(a) Find the velocity and acceleration at a point $(0,1,3)$ after 2 seconds for a 3 dimensional flow given by $u=y z+t, v=x z-t, w=x y$
(b) A circular plate of diameter 1.2 m is placed vertically in water in such a way that the centre of the plate is 2.5 m below the free surface of water. Determine
(i) Total pressure in the plate [3 marks]
(ii) Position of the centre of pressure
(c) When the pressure of a liquid is increased from $3.5 \mathrm{~N} / \mathrm{m}^{2}$ to $6.5 \mathrm{Mn} / \mathrm{m}^{2}$, its volume decreases by $0.08 \%$. Find the bulk modules of the elasticity of the liquid [3 marks]
(d) A clean tube of diameter 4 mm is immersed in a liquid with a coefficient of surface tension of $0.05 \mathrm{~N} / \mathrm{m}$. The angle of contact of the liquid with the glass is $140^{\circ}$. The density of the liquid is $13600 \mathrm{~kg} / \mathrm{m}^{3}$. Find the level of the liquid in the tube relative to the free surface of the liquid surface the tube.

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