CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

MATH 324: DYNAMICS

STREAMS: BSC (MATHS) **TIME: 2 HOURS**

DAY/DATE: FRIDAY 14/12/2018 **2.30 PM – 4.30 PM**

INSTRUCTIONS:

- **Answer Question One and any other Two Questions**
- **Adhere to instructions on the answer booklet**

QUESTION ONE (30 MARKS)

(a) A particle with velocity $v_0 = 2i + 4j$ at $t = 0,$ undergoes a constant acceleration $a = 3m/s$ at an angle $\theta = 130°$ from the positive direction of the x-axis. Find the particles velocity at $t = 5.0$ seconds. [4 marks]

(b) A particle moving with an initial velocity $v = 50m/s$ undergoes an acceleration $a = (35 + 2t^3)i - t^2 j$. Obtain the particles position and velocity after 3.0s assuming that its starts at the origin. [4 marks]

(c) Find the centre of mass of three particles at the vertices of an equilateral triangle given that the masses of the particles are 100g, 150g and 200g respectively and each side of the equilateral triangle is 0.5 m long assuming that one of the vertex is at the origin. [4 marks]

(d) Given the point $P(-2,6,3)$ and vector $A = y_{ax} + (x + z)a_y,$ express P and A in cylindrical and spherical coordinates. [4 marks]

(e) Find the torque of a force $7i+3j-5k$ about the origin given that the force acts on a particle whose position vector is $i-j+k$ [4 marks]

(f) Test for an extremum the functional

$$I[y(x)]=\int_0^1 (xy+y^2-2yy^2)dx, y(0)=1\ y(1)=2$$ [5 marks]

(g) Solve the Lagrange linear equation $yq-xp=z$ where [3 marks]

$$P=\frac{\partial z}{\partial x} \quad , \quad q=\frac{\partial z}{\partial y}$$

(h) State the Hamilton's principle for non-conservative system. [2 marks]

QUESTION TWO

(a) Find the equations of motion of a particle of unit mass moving on a plane in a conservative force field using Hamilton's principle. [10 marks]

(b) Prove that the shortest distance between two points is a long a straight line. [10 marks]

QUESTION THREE

(a) Find the shape of the curve of the given perimeter (P) enclosing maximum area. [10 marks]

(b) The launching speed of a projectile is five times the speed it has at its maximum height. Calculate the elevation angle at launching.

(c) A projectile is launched from ground level with speed V_0 at an angle θ_0 above the horizontal. Obtain

(i) The maximum height obtained by the projectile

(ii) The distance from the starting point at which the projectile strikes the ground (Range).

QUESTION FOUR

(a) Express the vector $B=\frac{10}{r}a_r+r\cos\theta a_\theta+a_\varnothing$ in cartesian and cylindrical coordinates and find $B=(-3,4,0)$ [10 marks]

(b) Two blocks of masses 10kg and 20kg are placed on the x-axis. The first mass is moved on the axis by a distance 2cm. By what distance should the second mass be moved to keep the position of the centre of mass unchanged. [4 marks]

(c) The angular speed of a motor wheel is increased from 1200 rpm to 3120 vpm in 1 seconds.

(i) Calculate its angular acceleration assuming the acceleration is uniform [3 marks]

(ii) Obtain the number of revolutions the engine makes during this time. [3 marks]

QUESTION FIVE

(a) Given the free particle Hamiltonian $H=\frac{p^2}{2m}$, obtain the Hamilton Jacobi equations and the Hamilton's characteristic functions (w) [4 marks]

(b) The position of an electron is given by $\vec{r}=3tc-4t^2j+2k$

(i) Find $v(t)at\ t=2$ seconds [2 marks]

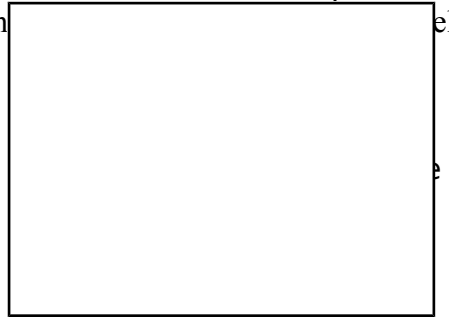
(c) A particle moves such that its position is $r=i+4t^2j+tk$. Obtain

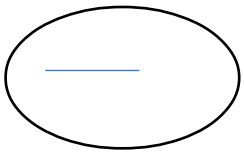
(i) Velocity $|v(t)|$ at $t=0$ [2 marks]

(ii) Acceleration $|a|$ [1 mark]

(d) Given the vector field $D=r\sin\phi a_r-\frac{1}{r}\sin\theta\cos\phi a_\theta+r^2a_\phi$, determine $D\ at\ P(10,150°,330°)$ [2 marks]

(c) A uniform disc of radius R is put over another uniform disc of radius 2R of the same thickn elow. Locate the centre of mass of the system. [3 marks]





(d) Solve the Lagrange linear equation $y^2 P - xyz = x(z - 2y), P = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

[6 marks]
