CHUKA   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (ARTS), BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF SCIENCE (MATHS), BACHELOR OF ARTS(ECON&MATHS) AND BACHELOR OF SCIENCE (ECON STAT)

MATH 322: ORDINARY DIFFERENTIAL EQUATIONS I

STREAMS: BED SCI, BED ARTS, BSC (MATH), BA (ECON & MATH), BSC (ECON & STAT)   **TIME: 2 HOURS**

DAY/DATE: TUESDAY 11/12/2018   **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS:
- **Answer question ONE and any other TWO questions.**

QUESTION ONE (COMPULSORY) (30 MARKS)

(a)   Define

   (i)   The order of a differential equation   (1 mark)

   (ii)   The degree of a differential equation   (1 mark)

(b)   Determine the order and degree of each of the following differential equations

   (i) $y'^3 - 4y = e^x$
            $y'' + 5$ (2 marks)

   (ii) $\dfrac{d^2 y}{dx^2}$
            $\dfrac{dy}{dx}^4 = 0$ (2 marks)
            $\dfrac{d^3 y}{dx^3} - 3$

(c)   Solve the differential equation

$$y'' - 4y' + 4y = 0 \quad \text{given} \quad y(0) = 1 \quad \text{and} \quad y(1) = e^2 \tag{4 marks}$$

(d)   Obtain the differential equation associated with the primitive
$$y = c_1 e^{-3x} + c_2 e^{x} + c_3 e^{-x} \tag{4 marks}$$

(e)   Solve the differential equation

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0 \tag{4 marks}$$

(f)   Solve the equation $\dfrac{1}{x}\dfrac{dy}{dx} + 4y = 2$ given $y(0) = 4$ (5 marks)

(g)   Solve $\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} = 5$ (4 marks)

(h)   Solve the differential equation

$$(x+1)\dfrac{dy}{dx} = x(1 + y^2) \tag{3 marks}$$

QUESTION TWO (20 MARKS)

(a)   Solve the homogeneous equation

$$\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{2xy} \tag{9 marks}$$

(b)   Solve the equation $y'' - 2y' + y = 3e^{4x}$ given that when $x = 0, y = \dfrac{-2}{3}$ and

$y' = 4\dfrac{1}{3}$. (11 marks)

QUESTION THREE (20 MARKS)

(a)   Solve the differential equation

$$\frac{dy}{dx}=\frac{x^3+y^3}{xy^2}$$

(4 marks)

(b)   Solve the differential equation

$$2\frac{d^2y}{dx^2}+3\frac{dy}{dx}-5y=6\sin 2x$$

(10 marks)

(c)   Solve the linear differential equation

$$\frac{dy}{dx}+1=\frac{-y}{x}, \quad y(1)=1$$

(6 marks)

QUESTION FOUR (20 MARKS)

(a)   Find the general solution of the equation

$$(x-2)\frac{dy}{dx}+3\frac{(x-1)}{(x+1)}y=1$$

(10 marks)

(b)   Given the boundary conditions that $y(-1)=5$, find the particular solution of (a) above.

(2 marks)

(c)   The population of Chuka University is known to satisfy the logistic law

$$\frac{dN}{dt}=\frac{1}{50}N(1000-N).$$ Prove that the population $N(t)=\frac{1000}{i+c\,e^{-20t}}$ where c is an arbitrary   constant.

(8 marks)

QUESTION FIVE (20 MARKS)

(a)   Find the general solution of the differential equation $x^2y''-2x\,y'+2y=x^3$

(8 marks)

(b)   Solve the differential equation
$$(y^2e^{xy^2}+4x^3)dx+(2xy\,e^{xy^2}-3y^2)dy=0$$

(4 marks)

(c)   Solve the equation

$$2y''-11y'+12y=3x-2$$

(8 marks)