CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 321: CALCULUS III

STREAMS: BSC (GEN) **TIME: 2 HOURS**

DAY/DATE: MONDAY 17/12/2018 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS: Answer question ONE (Compulsory) and any other TWO questions

QUESTION ONE (COMPULSORY) – 30 MARKS

(a) State the Rolle's theorem and verify for $f(x)=x^2+2x^{-8}$ on $[-4,2]$

[5 marks]

(b) Evaluate the following limits using L' Hopitals rule

(i) $\lim \text{¿} x \rightarrow \infty \frac{x^3+x+1}{3x^3+4}$

[3 marks]

(ii) $\lim \text{¿} x \rightarrow \infty \frac{\sin 3x}{x}$

[2 marks]

(c) Find the sum of the series [4 marks]

$$\sum_{k=1}^{\infty}\left(\frac{3}{4^k}-\frac{2}{5^{k-1}}\right)$$

(d) Find the volume of the solid under $f(x,y)=12-\frac{1}{2}x-\frac{1}{8}y$ over the rectangular $[0,8]x[0,16]$

[5 marks]

(e) Evaluate $\int_{1}^{4}\int_{-1}^{2}(2x+6x^{2}y)dydx$ [4 marks]

(f) Verify the Lagrange's mean value theorem for the function $f(x)=x^{\frac{2}{3}}$ in the interval $[-8,27]$

[3 marks]

(g) Use double integration to find the area of the triangle bound by $y=0, x=1 \wedge y=2x$

[4 marks]

QUESTION TWO (20 MARKS)

(a) Use the Maclaurin's theorem to expand $f(x)=\sqrt{1+x}$ and use it to approximate $\sqrt{1.01}$ to 5 decimal places [7 marks]

(b) Find the mass and centre of mass for a rectangular Lamina bounded by $y=x^{\frac{1}{2}}$, $y=0$ and $x=1$ having a mass density function $e(x,y)=x$ [9 marks]

(c) Show that $\lim ¿ x \to 2\left[\frac{2x^{2}-3x-2}{x-2}\right]=5$

[4 marks]

QUESTION THREE (20 MARKS)

(a) Apply the integral test to determine the convergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{n+1}$$

[4 marks]

(b) (i) If $f(x,y)=x^3y^2-2x^2y+3x$ find the second derivatives [3 marks]

(ii) Determine whether or not $f(x,y)=x^3y^2-2x^2y+3x$ is a harmonic function [3 marks]

(c) Find the surface areas of the surface $z=6-3x-2y$ above the region R bounded by

$$y=0, x=2 \wedge y=\frac{-3}{2}x+3$$ [5 marks]

(d) Evaluate $\int_0^1\int_0^2\left(x^2y+xy^3\right)dydx$ [5 marks]

QUESTION FOUR (20 MARKS)

(a) (i) State the Ratio test for the convergence of an infinite series $\sum_{n=0}^{\infty} a_n$ [2 marks]

(ii) Hence use ratio test to determine the convergence of the series

$$\sum_{n=1}^{\infty}\frac{2^n}{2n!}$$

[5 marks]

(b) Determine the volume V of the solid under the surface $Z=4-x^2-y$ and over the rectangle R given the $R=\{(x,y);0\le x\le 1, 0\le y\le 2\}$

[8 marks]

(c) Find the power series expansion for the function $f(x)=\left(1+x^2\right)^5\cos x$ [5 marks]

QUESTION FIVE (20 MARKS)

(a) Use the comparison test to determine the convergence of the series [5 marks]

$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$

[5 marks]

(b) Calculate the moments of the triangle bounded by the lines $y = x - 1, x = 0 \wedge y = 0$ having density $\mathrm{e}(x, y) = xy$ [9 marks]

(c) Find the Taylor series for $f(x) = Inx$ about $x = 1$ and use it to approximate the value of ¿1.1

[6 marks]
