

CHUKA



UNIVERSITY

UNIVERSITY EXAMINATIONS

EMBU CAMPUS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 304: COMPLEX ANALYSIS I

STREAMS: SB

TIME: 2 HOURS

DAY/DATE: TUESDAY 11/12/2018

11.30 A.M. – 1.30 P.M.

INSTRUCTIONS:

- Answer question ONE and any OTHER TWO.

Question one (30 marks)

a. If $z_1 = 2 + 4i$ and $z_2 = -3 + 8i$ find

i. $2z_1 - z_2$ (3 marks)

ii. $\bar{z}_2 z_1$ (3 marks)

iii. $z_1^{-1} + \bar{z}_2$ (3 marks)

b. Describe the set of point z in the complex plane satisfying the equation $z - i = 1$

(4mks)

c. Write the following complex numbers in polar form

i) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ (3 marks)

d. Show that the limit $\lim_{z \rightarrow 0} \frac{z}{\bar{z}}$ doesn't exist (5 marks)

e. Show that the complex function $f(z) = 2x^2 + y + i(y^2 - x)$ is not analytic at any point (4 marks)

f. Evaluate the integral $\oint_c \frac{dz}{z^2 + 1}$ where c is the circle $|z| = 4$ (5 marks)

Question two (20 marks)

i) Write $\frac{12}{\sqrt{3} + 1}$ in polar form (4 marks)

ii) Compute $\lim_{z \rightarrow i} \frac{(3+i)z^4 - z^2}{z+i}$ (5 marks)

iii) Evaluate $\oint_c \frac{e^z}{(z-2)(z-4)} dz$ around a circle $|z| = 5$ (4 marks)

iv) Find the poles and the corresponding residue of $f(z) = \frac{z^3 + 5z + 1}{z - 2}$ and the corresponding residue (6 marks)

Question three (20 marks)

i) $\text{Re}((1+i)z - 1) = 0$ (4 marks)

ii) Work out $(1+i)^{1/4}$ (6 marks)

iii) Show that the principal square root function $z^{1/2}$ is discontinuous at the point $z_0 = -1$ (6mks)

- iv) Show whether the polynomial $f(z) = z^2 + z$ is analytic for all z (4 marks)

Question four (20 marks)

- i) Find the real and the imaginary part of the complex number $\frac{z+1}{2z+5}$ where $z = x + iy$ (5 marks)

- ii) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the straight lines from $(0,3)$ to $(2,3)$ and then from $(2,3)$ to $(2,4)$ (10 marks)

- iii) Given that for $z = x + iy$ show that $z\bar{z} = |z|^2$ (5 marks)

