APPLICATION OF ASYMMETRIC-GARCH TYPE MODELS TO THE KENYAN EXCHANGE RATE AND BALANCE OF PAYMENTS OF TIME SERIES DATA

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A Thesis Submitted to the Graduate School in Partial Fulfilment of the Requirements for the Award of the Degree of Master in Applied Statistics of Chuka University

CHUKA UNIVERSITY

SEPTEMBER 2022

DECLARATION AND RECOMMENDATION

Declaration

This thesis is my own original work and has not been submitted before to any other institution for examination and award of any degree.

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Recommendation

This thesis has been examined, passed and submitted with our approval as University supervisors

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DEDICATION

I dedicate this thesis to my parents for their spiritual, social, and financial support during my years of education. May God Bless You.

ACKNOWLEDGEMENT

First, I thank Almighty God for the gift of good health and strength throughout the period of my studies. Special acknowledgement and gratitude go to my supervisors: Dr. Dennis K. Muriithi, Ph.D., and Dr. Adolphus Wagala, Ph.D., for substantial support and advice.

My sincere gratitude to my parents, sister, brothers and fellow students for they mutual co-existence throughout my studies.

Lastly, I express my utmost thanks to the Chuka University for awarding me the scholarship to pursue my career. I could not have made it without it. Thank you.

ABSTRACT

The critical concern of financial market investors is uncertainty of the returns. The symmetric-GARCH type models can capture volatility and leptokurtosis. However, they do not capture leverage effects, volatility clustering, and the thick tail nature of financial time series. The primary objective of this study was to apply the asymmetric-GARCH type models to Kenyan exchange and balance of payments of time series data to overcome the shortcomings of symmetric-GARCH type models. Secondary objectives included fitting asymmetric-GARCH type models to the Kenyan exchange rate and Balance of payments data, identifying the best asymmetric-GARCH type model(s) that best fit(s) the Kenyan exchange rate and Balance of payments data and forecasting the Kenyan exchange rate and Balance of payments data trends using the best asymmetric-GARCH type model. The study compared five asymmetric Conditional Heteroskedasticity class of models: IGARCH, TGARCH, APARCH, GJR-GARCH, and EGARCH. Monthly secondary data on the exchange rate from January 1993 to June 2021 and Balance of payments from August 1998 to June 2021 were obtained from the Central Bank of Kenya website. Asymmetric GARCH models were fitted to the stationary log-differenced data based on the functions in the RUGARCH package in R. The best fit model is determined based on minimum value of Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC). The optimal variance equation for the exchange rates data was APARCH (1,1) - ARMA (3,0) model with a skewed normal distribution (AIC = -4.6871, BIC = -4.5860) since it accounts for leverage and the Taylor effect. The optimal variance equation for the Balance of payment data was ARMA (1,1) - IGARCH (1,1) model with a skewed normal distribution (AIC = -0.14475, BIC = -0.07882) due to absence of (persistent) volatility clustering in the series. Volatility clustering was present in exchange rate data. Both series did not show evidence of leverage effect. Estimated Kenya’s exchange rate volatility narrows over time, indicating sustained exchange rate stability. While the balance of payment volatility has narrowed over time, the balance of payment deficit keeps widening. Thus, the government should take measures to ensure that it maintains it competitiveness in the global market to attract foreign direct investment and promote exports of goods and services.

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LIST OF ABBREVIATIONS AND ACRONYMS

ACF	Autocorrelation Function
AIC	Akaike Information Criterion
AR	Autoregressive
ARCH	Autoregressive Conditional Heteroscedasticity
ARIMA	Autoregressive Moving Average
ASE	Amman Stock Exchange
BoP	Balance of Payments
BIC	Bayesian Information Criterion
CAB	Current Account Balance
CBK	Central Bank of Kenya
COVID-19	Coronavirus Disease 2019
DEG	Ding, Granger and Engle
ER	Exchange Rate
EACF	Extended Autocorrelation Function.
EGARCH	Exponential Generalized Autoregressive Conditional Heteroscedasticity
EMU	European Monetary Unity
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GJR-GARCH	Glosten, Jagannathan and Runkle Generalized Autoregressive Conditional Heteroscedasticity
GMM	Generalized Method Moments
HQIC	Hannan-Quinn Information Criterion
IGARCH	Integrated Generalized Autoregressive Conditional Heteroscedasticity
LM	Lagrange Multiplier
MA	Moving Average
ME	Mean Error
NSBT	Negative Sign Bias Test
PACF	Partial Autocorrelation Function
PGARCH	Power Generalized Autoregressive Conditional Heteroscedasticity
PSBT	Positive Sign Bias Tests
QMLE	quasi-MLE
RER	Real Exchange Rate

RMAE	Root Mean Absolute error
SBT	Sign Bias Test
SIC	Schwarz information criterion
TGARCH	Tension Generalized Autoregressive Conditional Heteroscedasticity
TIC	Theil Inequality Coefficient
US	United States of America
VaR	Value at Risk

CHAPTER ONE
INTRODUCTION

1.1 Background Information

Financial investors are always concerned about the uncertainty of the returns which can be driven by the price changes, market risks, and business performance instability (Alexander, 1999). Volatility has been used to proxy uncertainty. Volatility refers to the degree of fluctuations of a given phenomenon over time (Coondoo & Mukherjee, 2004). In financial markets, volatility is commonly defined as "the (instantaneous) standard deviation of the random Wiener-driven component in a continuous-time diffusion model" (Andersen, *et al.*, 2006). For instance, if asset returns have large swings, it has higher volatility. In turn, common volatility models relate to the "the conditional variance" of the underlying series (Tsay, 2010. Volatility is essential portfolio optimization, risk management, and asset pricing. As such, modelling and forecasting volatility of a financial time series is essential.

Measuring and quantifying risks in financial markets is usually a great challenge due to systematic risks (natural disasters, wars, inflation and interest rates fluctuations) that affect the entire market. Most financial time series possess volatility and have unique features referred to as stylized financial time series facts, which include the absence of autocorrelations, heavy tails, asymmetry in time scales, volatility clustering, and leverage effect (Sewell, 2011). Linear time series models do not adequately describe time series data that exhibit volatility since they assume the existence of linear dependence in given series (Akpan *et al.*, 2016). Furthermore, the linear models are built on the homoscedastic assumption which may not necessarily hold in real time-series data due to existence of trend or cyclical components. Time series data like exchange rate usually exhibit volatility clustering resulting in the violation of the homoscedastic assumption of the equality of variance over time (Cont, 2007). Therefore, non-linear models, for example, symmetric and asymmetric-GARCH type models, that capture market volatility, have been proposed as suitable models.

Symmetric properties of financial time series modelling were introduced by Engle and Nelson (1990). He developed the ARCH(p) which model the effect of conditional heteroskedasticity and serial correlation effect and the GARCH (p, q) model where the restricted variance is stated as a function of constant volatility and variance terms. The symmetric-GARCH models capture leptokurtosis and volatility clustering properties. The symmetric-GARCH type model fails to model the leverage effect property, a situation when an unanticipated reduction in prices increases anticipated volatility more than an unanticipated growth in the price of similar magnitude (Zakoian & Francq, 2010). Besides, symmetric GARCH type models do not always fully embrace the thick tails property of high-frequency economic time series. To overcome these problems, the Student's t-distribution as used by Bollerslev (1987), Baillie and Bollerslev (1989), Beine et al. (2002), and Fernandez and Steel (1998) has been used as a substitute to the normal distribution to fit the volatility since it has zero skewness and excess kurtosis.

Studies criticize the symmetric GARCH model since the magnitude of change only influences the restricted variance. That is, both past negative and positive fluctuations have the same impact on the current volatility. Since the conditional variance must be nonnegative, the parameters are often constrained to be nonnegative (Cryer & Chan, 2008). Thus, symmetric GARCH models do not capture the asymmetry effect in financial time series returns data. Asymmetry infers that the unanticipated bad news increases the restricted volatility more than the unanticipated good news of similar magnitude (Olweny & Omondi, 2011). Conversely, asymmetric-GARCH type models have the restricted variance only depending on the magnitude or size and not the sign of the shock (Engle & Bollerslev, 1986).

To address the shortcoming of symmetric GARCH models, asymmetric GARCH type models, including the Exponential GARCH (EGARCH) model by Nelson (1991), Glosten, Jagannathan, & Runkle GARCH (GJR-GARCH) model by Glosten et al. (1993) and the Asymmetric Power ARCH (APARCH) model by Ding et al. (1993), the Threshold GARCH (TGARCH), and the Integrated GARCH (IGARCH) model by Engle and Bollerslev (1986) are more suitable models because they capture the time-varying variance of such time series (Moffat *et al*., 2017). These models have unique innovations that capture the asymmetric effect in financial time series. For instance, the EGARCH model uses the natural logarithm to model the restricted variance unlike the GARCH model, which models the variance directly. Thus, the parameter boundaries are not required to guarantee a positive restricted variance. The asymmetric-GARCH type models tolerate asymmetric effects of positive and negative innovation (Hafner & Linton, 2017).

Most empirical studies have demonstrated that asymmetric GARCH type models are more robust compared to the symmetric GARCH type models. The stock market has been a common area where the volatility models have been useful. For instance, Liu & Brorsen (1995) used an asymmetric model to capture the skewness effect of deutschmark returns. Petrică & Stancu (2017) empirically examined how symmetric (ARCH and GARCH) and the asymmetric-GARCH type models (EGARCH, TGARCH, and PARCH) could capture the volatility of daily returns of EUR/RON exchange rates. They found out that all the asymmetric models were better than the standard ARCH in minimising the volatility prediction errors. The best model for the series was EGARCH (2,1), assuming the student's t distribution.

The asymmetric and symmetric GARCH models have also been used to examine the volatility of the inflation rates. Nortey *et al*. (2014) compared the standard ARCH, GARCH, and EGARCH model using Ghana's monthly inflation from 2000 January to 2013 December. The results illustrated that asymmetric models (EGARCH) outperformed the standard ARCH and GARCH models. The EGARCH (1, 2) model with the mean equation of $ARIMA\ (3,1,2) \times (0,0,0)_{12}$ being the best fit model for the data outperforming other competing ARCH, GARCH, and EGARCH models.

Hasbalrasol, Kandora, & Hamdi (2017) examined volatility models' accuracy and predictive performance for the monthly Sudanese exchange rate (SDG/USD) return data from 1999 January to 2013 December. They compared the standard GARCH, Asymmetric GARCH, and ARMA models assuming the non-normal and normal Student distributions. The findings revealed that the asymmetric GARCH type models provided a better fit for the Sudanese exchange rate assuming the non-normal distribution and improved the restricted variance forecasts than the GARCH model. The Ding, Granger and Engle (DEG)-GARCH model assuming the student t-distribution {AIC = -7.844, Bayesian Information Criterion (BIC) = -7.665} was the best fit for the series. The model produced reliable forecasts and adequately estimated the Sudanese pound exchange rate volatility. Besides, the leverage effect in the series was a common stylised fact in most financial series.

In Kenyan, Fwaga, Orwa, & Athiany (2017) compared competing orders of the standard GARCH models $EGARCH\ (1,1)$ using Kenyan monthly inflation rate data from 1990 January to 2015 December. The study findings revealed that the EGARCH (1,1) model was best model fit for forecasting inflation outperforming $GARCH\,(1,1)$, $GARCH\ (1,2), GARCH\ (2,1)$, and $GARCH\ (2,2)$. Wagala et al. (2012) examined the most efficient model from the symmetric {ARCH(q) and GARCH (p, q)} and the asymmetric GARCH {IGARCH (p, q), EGARCH (p, q), and TGARCH (p, q)} models fitted the Nairobi Securities Exchange weekly returns series. Based on the minimisation of Shwartz Bayesian Criteria (SBC), Akaike Information Criteria (AIC) and the Mean Squared Error (MSE), the study findings revealed that the IGARCH model, assuming student's t-distribution was the best model for modelling and forecasting volatility of the Nairobi's Stock market series. Overall, the few reviewed studies have a common agreement that asymmetric are better than symmetric-GARCH models. Thus, the current study applied asymmetric-GARCH models to exchange rate and Balance of Payments (BoP) data.

An exchange rate is the value of a country's currency versus the currency of another country or economic zone so-called common currency area (Mishkin & Eakins, 2009). Countries are now operating on a floating exchange rate regime, where exchange rate changes are driven by market supply and demand. Before 1972, most countries globally were on a fixed exchange rate regime where currencies had a fixed rate relative to the United States of America (US) dollar (Aristotelous, 2002). For instance, Kenya maintained a fixed exchange rate regime whose adjustment was based on key factors such as export earnings, import payments, tourist incentives, and external public debt from 1966 to early 1971 (Mwamadzingo, 1988). The external value of the Kenya shilling was valued in various standards at different points in time. Since then, Kenya's external value of the Kenya shilling has been fluctuating as determined by the supply and demand forces in international trading activities such as importation and purchase of government bills, particularly from developed countries.

Exchange rate fluctuations affect a given economy's economic performance by impacting gross domestic product growth and inflation (Onyancha, 2012). Schnabol (2007) examined the influence of exchange rate volatility on economic growth on forty-one small open economies in the European Monetary Unity (EMU) region from 1994 to 2005. The author established that exchange rate volatility negatively influences economic growth and recommended that macroeconomic stability is essential to maintain positive economic growth. An exchange rate fluctuation can also lead to inflation. Exchange rate appreciation makes exporters lose their competitiveness in the international market, hence reducing their sales. In turn, it worsens the balance of payments of their home country. As such, changes in the exchange rate have a significant impact on a country's balance of payments (BoP) (Ndung'u, 2016). Therefore, the monetary authorities attach much importance to properly managing a country's foreign exchange.

The BoP is a summary statement of a country’s transactions with the rest of the countries globally via trade and finance. BoP is divided into two components: the current account and capital account. The current account records all trade of goods and services, factor income receipts and payments, and net current transfers whereas the capital account records purchase and sales of financial assets (Wanjau, 2014). Notably. inflows in the capital account finance the current account; such that the BoP is balanced. An external balance is reached when both accounts offset each other with no surplus or deficit in the BoP (Calvo, 2000).

Asymmetric GARCH models have also been established to capture BoP volatility well due to evident of non-linear dependence in the series. Tang (2009) applied an AR(p) model to the balancing item series (net errors and omissions) of the BoP of 20 industrial countries, including the United States, Germany, and the United Kingdom, to remove any linear structure. The non-linearity tests indicated the existence of non-linear dependencies for 16 of the 20 countries. Therefore, the authors recommended that the non-linear dynamics of these balancing items of the BoP ought to be incorporated when modelling and forecasting.

Tang & Hooy (2007) established that the volatility of the Australian BoP is better fitted in an Asymmetric-Component GARCH model with a resultant long memory in absorbing shocks. Hakim & Sriyana (2020 estimated the Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model for Indonesia's annual Current Account Balance of Payments (CAB) data from 1985 to 2018 and established evidence of volatility in the CAB. The resulting conditional value at risk (VaR) or standard deviation indicated that the Indonesian CAB is stable. Therefore, the current study also sought to establish the best fit Asymmetric GARCH model for Kenya's BoP data and, as a result, establish its volatility over time.

1.2 Statement of the Problem

An exchange rate fluctuation affects the economic performance of a given economy as it impacts output growth and price inflation. Therefore, exporters will lose competitiveness in the global marketplace when the exchange rate grows. The transactions and earnings of exporters will diminish, worsening the balance of payments. The symmetric-GARCH type of models has been used to model exchange rate and balance of payment. The symmetric ARCH and GARCH models capture leptokurtosis and volatility clustering effects. However, some studies criticise the symmetric-GARCH model since the restricted variance only depends on the magnitude of change. They assume a symmetric distribution hence fail to capture the leverage effect property in financial time series which states that an unanticipated price reduction increases anticipated volatility more than an unanticipated increase in a price of similar size or magnitude. Also, when using symmetric-GARCH type models, they do not fully embrace the thick tails property of high occurrence financial time series. Many asymmetric GARCH type models, such as the EGARCH, GJR-GARCH model, TGARCH, IGARCH, and APARCH models have been suggested to address the shortcomings of symmetric models. Therefore, the study examined the most efficient asymmetric-GARCH type models to Kenya's exchange rate and balance of payment to overcome shortcomings of symmetric-GARCH type models.

1.3 Objectives of the Study

1.3.1 General Objectives

This study applies the asymmetric-GARCH type models to the Kenyan exchange rate and balance of payments.

1.3.2 Specific Objectives

The following specific objectives guided the study:

i. To fit asymmetric-GARCH type models (EGARCH, IGARCH, APARCH, GJR-GARCH, and TGARCH) to the Kenyan exchange rate and BoP data.
ii. To identify the best asymmetric-GARCH type model(s) that best fit(s) the Kenyan exchange rate and BoP data.
iii. To forecast the Kenyan exchange rate and BoP data trends using the best asymmetric-GARCH type model.

1.4 Research Questions

i. How do asymmetric-GARCH type models (EGARCH, IGARCH, APARCH, GJR-GARCH, and TGARCH) fit the Kenyan exchange rate and BoP data?
ii. Which of the asymmetric-GARCH type models is best for forecasting Kenyan exchange rate and BoP data?
iii. What are the future trends of Kenyan exchange rate and BoP data forecasted using the best asymmetric-GARCH models?

1.5 Significance of the Study

The study adds to the existing knowledge on the asymmetric-GARCH type models. By using exchange rates and BoP data the study demonstrates how the theoretical underpinning of the models manifest in series with different time series properties. Empirically, the research findings can help government policy makers understand the Kenya's exchange rate and BoP volatility. The findings from the study also help policy makers redefine strategies that ensure a stable exchange rate and BoP by examining their volatilities. Commercial Banks also need to hedge foreign exchange, which increases when a bank holds resources or liabilities in external currencies and influences the investment earnings of the bank. Thus, understanding the predictability of exchange rates and BoP may enable the government, through the central bank, to mediate in the market in managing the two to a sustainable level when the need arises. The recommendation for future research would also help researchers and academicians to carry out more studies to extend the understanding of how exchange rate volatility influences the stability of payments in Kenya.

1.6 Scope of the Study

The asymmetric conditional heteroscedastic class of models compared are the EGARCH, TGARCH, IGARCH, APARCH, and GJR-GARCH. The models are particularly applied to monthly exchange rate data spanning from January 1993 to June 2021 and the BoP data spanned from August 1998 to June 2021. The period is suitable since Kenya is already on a flexible exchange rates regime. The best fit model is determined based on parsimony (AIC, BIC, Log-Likelihood criterion) and minimisation of prediction production errors (Mean error [ME] and Root Mean Absolute error [RMAE]). A 12-month step ahead forecast horizon of volatility of the two series made with unconditional 1-sigma confidence bands.

1.7 Operational Definition of Terms

Asymmetric Models: These are models used to capture the asymmetric features of volatility. It assumes that shocks of the equal magnitude (positive or negative) have a different result on volatility.

Balance of Payments: is the country's transactions summary of international trade and financial transactions made by its residents.

Exchange Rate: is the value of a county's currency in terms of another currency as in the current study, US Dollar/Kenya shillings (US/Ksh).

Leptokurtic: More peaked distribution with fat tails, and a thin midrange than a standard normal distribution.

Leverage Effect: Fluctuations in stock values tend to be negatively correlated with fluctuations in volatility; that is, the value of economic assets often reacts more noticeably to bad news than good news.

Mesokurtic: A distribution with a kurtosis of zero or practically less than 3. The distribution is moderate in breadth and curves with a medium peaked height.

Symmetric Models: The restricted variance depends on the size and not the sign of the causal shock.

Volatility: refers to the degree of fluctuations of a series over time.

Volatility Clustering: Is the observation that large (small) fluctuations follow large (small) fluctuations in absolute terms.

CHAPTER TWO

LITERATURE REVIEW

2.1 Exchange Rate and Balance of Payments

The current section previews the economic perspective of exchange rates and Balance of Payments

2.1.1 Exchange Rate

An exchange rate is the value of one cash in terms of another currency (Mishkin & Eakins, 2009). From 1972, most countries worldwide used a fixed exchange rate system where the nations had a fixed rate relative to the US dollar. Kenya's economy was under an independent float from 1992 to 1997 and later shifted to a managed float since 1998. This is usually determined by the demand for currency in international trade, such as importers and government bills. Acceptance of the floating exchange rate system was anticipated to benefit Kenya (Ndung'u & Mwega 1999). This is because it allows an automatic adjustment of the exchange rate guided by the invisible hand of the demand and supply of foreign exchange. Besides, it allows Kenya the freedom to engage in its financial policy without regard to the impact of the BoP. The expansion of world trade and investment volatility has made the exchange rate a vital determinant of trade profitability and the country's balance of payments (Kim, 2003). The exchange rate stabilization is usually directed towards achieving two major objectives: stabilizing inflation and achieving export competitiveness.

The exchange rate volatility concept has been an area of concern since it largely affects international trade. Past literature has demonstrated that exchange rate volatility affects the level of exports with no consensus on the direction of effect since volatility is bidirectional. Munyama & Todani (2005) evaluated exchange rate volatility using the moving average standard deviation and GARCH (1, 1) and positively associated with export performance. In another study, Kasman & Kasman (2005) established a positive impact of exchange rate volatility on trade, in the long run, using Cointegration and error correction models. On the contrary, studies such as Esquivel and Felipe (2002) and Doganlar (2002) have established a negative association between exchange rate volatility and exports. Therefore, the current study proposes asymmetric GARCH models as a potential model to capture exchange rate volatility.

2.1.2 Balance of Payment

BoP is the value that nations use to track global financial transactions at a particular period (Heakel, 2008). The BoP summarises a nation's transactions with its trading and financing partners across the globe. The BoP have two parts; namely' the current account (that records the imports and exports of goods and services, receipts and payments, and gross current transfers) and the capital account (that records all the financial assets transactions). Therefore, the two records are expected to balance, hence the name of the BoP. A deficit in the current account is an equivalent surplus in the capital account, thus sums up to zero. A state of external equilibrium is attained when the current and financial accounts are equal; that is, there is no surplus or deficit in either record. A surplus in the BoP arises when the value of a nation's exports of goods and services, income, and current transfers from other nations surpass its payments for imports of goods and services, payments, and remittances abroad. The opposite is true when it comes to a deficit in BoP. Factors that influence the BoP include terms of trade, domestic money supply, exchange rate, domestic investment, inflation, foreign direct investment, external borrowing, and remittances (Fischer, 1993).

Under the flexible exchange rate regime, demand and supply forces for foreign exchange were anticipated to increase exports prices, which fosters the country's export sector. Exchange rate stabilization anchors domestic prices more than a managed float (Adam, 2012). Conversely, a floating rate ensures efficient adjustment of an economy to external shocks, maintaining macroeconomic stability. However, it can boost capital flight and discourage foreign investment leading to an unstable macroeconomic environment (Aguirre & Calderón, 2005). Exchange rate instability influences the international competitiveness of local firms since it affects input and output prices (Joseph, 2002) and the global competitiveness of a country due to the potential loss of value of money. For instance, exporters lose competitiveness in the international market due to reduced profits due to exchange rate appreciation. Conversely, importers' competitiveness increase in the domestic market, leading to excess imports. In any case, exchange rate fluctuations worsen the BoPs. Depreciation of currency discourages imports, raising import prices relative to domestic prices. Thus, exporters will play a competitive advantage against exporters from other nations, increasing their export levels (Yau & Nieh, 2006). Therefore, the current account deficit is a structural problem arising from exchange appreciation which encourages imports while diminishing exports. In conclusion, it is vital to understand the volatility of exchange rates since it benefits players in international trade and the economy. Thus, the current study proposes asymmetric GARCH models as a potential model to capture the spill over effect of exchange rate volatility on international trade.

2.2 ARMA Model

Let X_t be the observed time series and further let $e_t's$ be a sequence of independent identically distributed with a mean zero and variance δ_e^2 , an autoregressive process of order p {AR(p)}, so-called Yule–Walker equations, attributed to the works of Yule (1925; 1926) satisfies the equation 2.1 with e_t being independent of $X_{t-1}, X_{t-2}, \dots, X_{t-p}$

$$X_t = \emptyset_1 X_{t-1} + \emptyset_2 X_{t-2} + \emptyset_3 X_{t-3} + \cdots + \emptyset_p X_{t-p} + e_t \tag{2.1}$$

Equation 2.1 denotes X_t as a linear combination of the p recent observations of itself with the residual component, e_t not accounted for by the past values. Slutsky (1927) & Wold (1938) introduced the Moving Average (MA) process expressed in equation 2.2

$$X_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3} - \cdots - \theta_q e_{t-q} \tag{2.2}$$

Unlike in AR(p) process, the series is linearly depended on the present and past values of the stochastic term.

Later on, the ARIMA model was developed in the 1970s by George Box and Jenkins; hence Box and Jenkins methodology (Box, and Jenkins, 1970). The ARMA (p, q) process, has the MA (q) and AR (p) parts. An innovation of the two models is the autoregressive moving-average (ARMA) model brought forth by Box, Jenkins, and Reinsel (1994). The parametrization of the ARMA process combines the functional form of AR and MA models, as an amalgamation of both processes, to reduce the number of parameters used. It forms the mean equation since it gives a flexible and parsimonious estimation of conditional mean dynamics.

An ARMA (p, q) process is expressed as in equation 2.3.

$$X_t - \emptyset_1 X_{t-1} - \cdots - \emptyset_p X_{t-p} = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_q e_{t-q} \tag{2.3}$$

Where $e_t \sim N(0, \delta^2)$,

Using the backshift operator, we have the model for ARMA being

$$\left(1 - \emptyset_1 B - \emptyset_2 B^2 - \cdots - \emptyset_p B^p\right) X_t = (1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q) e_t$$

$$\emptyset(B) X_t = \theta(B) e_t \qquad (2.4)$$

From the above equation when $\emptyset(B) = 1$ then ARMA (p, q)≈ MA (q) and also when $\theta(B) = 1$ then the ARMA (p, q) ≈ AR (p). The optimal order for the ARMA (p, q) model can be selected using the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF). The ARMA model, with AR (p) and MA (q) terms, depicts the GARCH model's conditional mean of the exchange rate return series. The box- Jenkins modeling involved a preliminary analysis and an iterative three-stage process which are:

i. Model-identification. This stage involves selecting the order p and q of the AR and MA polynomial. The selection of p and q comes from the ACF and PACF functions.
ii. Model estimation. In this stage, the model's parameter estimation is considered by using various model estimation methods.
iii. Diagnostic-checking. In this stage, the model is examined for its adequacy in forecasting.
iv. Forecasting.

ARMA Estimation

The ARMA (p, q) process parameters can be estimated using conditional or maximum likelihood methods. The Maximum likelihood estimation (MLE) method is common. Like other time series models, the post-diagnostic checks in ARIMA modelling include testing if the model residues are white noise or have no ARCH effects. The Ljung–Box statistics is commonly used to examine the model’s adequacy. A correctly specified model, then the Ljung statistic ($Q(m)$) follows a Chi-squared distribution with $(m - g)$ degrees of freedom, where g is the number of model parameters.

2.3 Volatility Models

Financial markets respond to political upheavals, economic crises, wars, or natural disasters. Such events lead to higher volatility in the financial series. Financial series such as asset returns possess distinct features commonly referred to as stylized facts of financial data. The stylized facts include Serial dependence, Volatility changes over time, asymmetry, or heavy-tailed distribution with excess kurtosis. Statistically speaking, the conditional variance for given past observations expressed in equation 2.5 is non-constant over time, and X_t is conditionally heteroskedastic.

$$\text{Var}\,(X_t \mid X_{t-1}, X_{t-2}, \ldots.) \qquad (2.5)$$

The volatility can also be captured as the square root of the conditional variance (equation 2.6)

$$\delta = \sqrt{var\ (X_t|X_{t-1}, X_{t-2}, \ldots)} \qquad (2.6)$$

Since X_t is conditionally heteroskedastic, the assumption of Gaussian distribution and stationarity may not hold with most financial time series. Engle proposed autoregressive conditionally heteroskedastic- ARCH since the conditional variance is not constant over time and is autoregressive in nature. Bollerslev (1986) introduced the GARCH model as an innovation from the ARCH model. The following discusses the two forms of GARCH models.

2.3.1 Symmetric-GARCH Models

The symmetric property assumes that volatility increases more following adverse than positive shocks of the same magnitude. The generalized ARCH (GARCH) model captures such aspects as a conditional heteroscedastic model proposed by Bollerslev (1986) as a helpful extension of the ARCH model. Engle (1982) noted that the conditional variance did not depend on conventional econometric models' past. He thus proposed a model that captures the perception that current variance depends on its past and the non-constant nature of the one-period forecast variance. Besides, the model parameter should satisfy the non-negativity constrain and stationary assumptions.

Symmetric-GARCH models assume that positive and negative shocks have the same effect on volatility since it relies on squared residuals. In reality, especially in financial series such as the price of assets, the bad news is more pronounced than good news. In such a case, negative shocks cause higher volatility than positive ones, albeit being of the same size. The phenomenon has been termed the leverage effect. Ascribed to Brooks (2008), the term “leverage effect” stems from the perception that a stock's volatility tends to increase when returns are negative. To overcome such a shortcoming of symmetric models, asymmetric- GARCH models such as EGARCH have been developed.

2.3.1.1 ARCH (p) Model

Consider X_t is a stationary time series with $X_t = \delta_t \varepsilon_t$ where $\delta_t \geq 0$ and is generated by $X_{t-k}, k \geq 1$ and ε_t signifies the randomly distributed and independent variables with mean zero and variance of one.

As postulated by Engle (1982), ARCH is a function of past squared returns with heteroscedastic and volatility clustering properties. ARCH(q) model can be expressed as in equation 2.7

$$\delta_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 \qquad (2.7)$$

Where $\omega > 0, \alpha_i \geq 0,$ for $i = 1, \ldots, q$ and $\sum_{i=1}^{q} \alpha_i < 1$

If the term $\sum_{i=1}^{q} \alpha_i < 1$, then it is weekly stationary and the unconditional variance is defined as in equation 2.8.

$$E(\varepsilon_t^2) = \frac{\omega}{1 - \alpha_1 - \alpha_2 - \cdots - \alpha_q} \qquad (2.8)$$

Nonetheless, the ARCH model has shortcomings. It assumes that positive and negative shocks have the same impact on volatility since it relies on the previous squared shocks. The ARCH model is also restrictive, and the constraints become complicated for higher orders limiting its ability with the Gaussian process to capture excess kurtosis. Besides, the ARCH models can over fit the volatility since they are less responsive to large shocks (Engle, 1982). Owing to the shortcoming of ARCH models, an advanced model was brought forth: the generalized ARCH model.

2.3.1.2 GARCH (p, q) Model

This model was introduced by Bollerslev (1986). Let (z_t) be a sequence of $i.i.d.$ random variables, thus $Z_t \sim$ N (0, 1). (a_t) is the generalized autoregressive conditional heteroskedastic or GARCH (q, p) process if $a_t = \delta_t z_t$, $\forall\, t \in Z.$ δ_t is a nonnegative process such that;

$$\delta_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, a_{t-i}^2 + \sum_{j=1}^{p} \beta_j\, \delta_{t-j}^2 \tag{2.9}$$

And $\alpha_0 > 0, \alpha_i \geq 0,\ i = 1,2,\ldots,q, \beta_j \geq o, j = 1,2,\ldots,p$

The parameter restrictions guarantee the positivity of the conditional variance. In terms of the lag-operator, B, equation 2.9 results to equation 2.10.

$$\delta_t^2 = \alpha_0 + \alpha(B)a_t^2 + \beta(B)\delta_t^2 \tag{2.10}$$

Where $\alpha(B) = \alpha_1 B + \alpha_2 B^2 + \cdots + \alpha_q B^q$ and $\beta(B) = \beta_1 B + \beta_2 B^2 + \cdots + \beta_p B^p$ and $\alpha_i + \beta_i = 1$.

If the roots of the characteristic equation, i.e., 1 -$\beta_1 a$- $\beta_2 a^2 - \cdots - \beta_p a^p$ = 0 lie outside the unit circle and Z_t is stationary, then we have

$$\delta_t^2 = \frac{\alpha_0}{1-\beta(B)} + \frac{\alpha(B)}{1-\beta(B)} a_t^2 \tag{2.11}$$

$$\delta_t^2 = \alpha_0^* + \sum_{i=1}^{\infty} \delta_i\, a_{t-i}^2 \tag{2.12}$$

Where $\alpha_0^* = \frac{\alpha_0}{1-\beta(B)}$ and δ_i are coefficients of B_i in the expansion of $\alpha(B)[1-\beta(B)]^{-1}$

Bollerslev (1986) give the following properties of the GARCH model:

Mean

From equation (2.13), the conditional expectation and variance of a_t is

$$E(a_t) = E(\delta_t z_t) = \delta_t\, E(z_t\,) = \delta_t(0) = 0 \tag{2.13}$$

Since the expectation of Z_t is 0 and iid with a mean zero.

Second Moments or Variance

From equation 2.14, the variance of a_t^2 is given by

$$E(a_t^2)\ = [\delta_t^2\, z_t^2] = E(\delta_t^2)\, E(z_t^2) = E(\delta_t^2) \tag{2.14}$$

Since $E(z_t^2)$ =1 because it is a normal distribution with mean 0 and variance of 1, taking expectations on both sides of equation 2.14, we have equation 2.15.

$$E(\delta_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, E(a_{t-i}^2) + \sum_{j=1}^{p} \beta_j\, E(\delta_{t-j}^2) \tag{2.15}$$

Since $E(a_t^2)$ $=E(\delta_t^2) = E(a_{t-i}^2)$, under normality assumptions, the variance is estimated using equation 2.16;

$$E(a_t^2) = E(\delta_t^2) = \alpha_0 /[1 - (\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j)] \qquad (2.16)$$

GARCH (1, 1), The variance is given by the equation below

$$E(\delta_t^2) = \frac{\alpha_0}{1 - \alpha_1 - \beta_1} \qquad (2.17)$$

The Kurtosis

The first fourth moment of the time series is obtained using equation 2.18.

$$E(a_t^2) = E\{(\delta_t^2)^2 z_t^4\} = E\{(\delta_t^2)^2\} = E(z_t^4) = 3E\{(\delta_t^2)^2\} \qquad (2.18)$$

Since the fourth moment of a normal distribution is three, i.e.,$E(z_t^4) = 3$

But

$$E\{(\delta_t^2)^2\} = E\left\{(\alpha_0 + \sum_{i=1}^{q} \alpha_i\, a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \delta_{t-j}^2)^2\right\} = \qquad (2.19)$$

By expansion, equation 2.19 results in equation 2.20

$$E\ ((\delta_t^2)^2) = \alpha_0^2 + 2\alpha_0 \sum_{i=1}^{q} \alpha_i\, E(a_{t-i}^2) + 2\alpha_0 \sum_{J=1}^{P} \beta_j\, E\big(\delta_{t-j}^2\big) + \sum_{i=1}^{q} \alpha_i^2\, E(a_{t-i}^4) +$$

$$\sum_{j=1}^{p} \beta_j^2 E\big[(\delta_{t-j}^2)^2\big] + 2 \sum_{i=1}^{q} \sum_{j=1}^{p} \alpha_i \beta_j E(a_{t-i}^2 \delta_{t-j}^2) \qquad (2.20)$$

When we consider the case for $i = j = 1$ we get the GARCH (1, 1) model as

$$E\{(\delta_t^2)^2\} = \alpha_0^2 + \alpha_1^2 E(a_{t-1}^4) + \beta_1^2 E[(\delta_{t-1}^2)^2] + 2\alpha_1\beta_1 E(a_{t-1}^2 \delta_{t-1}^2)$$
$$+ 2\alpha_0^2 \alpha_1^2\, E(a_{t-1}^2) + 2\alpha_0\, \beta_1 E(\delta_{t-1}^2)$$

By simplifications and taking the right terms together, we have equation 2.21

$$E\{(\delta_t^2)^2\} = \alpha_0^2 + (3\alpha_1^2 + 2\alpha_1\beta_1 + \beta_1^2)\, E\{(\delta_{t-1}^2)^2\} + 2\alpha_0(\alpha_1 + \beta_1)\, E(\delta_{t-1}^2) \quad (2.21)$$

Assuming the process is stationary

$$E\{(\delta_t^2)^2\} = E\{(\delta_{t-1}^2)^2\}$$

Hence

$$E\{(\delta_t^2)^2\} = \alpha_0^2 + 2\alpha_0(\alpha_1 + \beta_1)\, E(\delta_{t-1}^2)/(1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2)$$

$$E\{(\delta_t^2)^2\} = \alpha_0^2 + 2\alpha_0^2(\alpha_1 + \beta_1)/(1 - \alpha_1 - \beta_1)(1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2)^{-1}$$

From $E(a_t^4) = 3E\{(\delta_t^2)^2\}$

$$E\{(\delta_t^2)^2\} = 3 = \alpha_0^2 \frac{[1 + 2(\alpha_1 + \beta_1)]}{\{ (1 - \alpha_1 - \beta_1)(1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2)^1\}} \quad (2.22)$$

Equation 2.22 is the fourth moment for GARCH (1, 1) model. The Kurtosis is given by equation 2.23.

$$K = \frac{E(a_t^4)}{[E(a_t^2)]^2} \quad (2.23)$$

Substituting for equation 2.23 by using equation 2.12 and equation 2.17, equation 2.24 is obtained as an estimate of kurtosis.

$$K = 3\frac{1 - (\alpha_1 + \beta_1)^2}{1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2} > 3 \quad (2.24)$$

which is strictly greater than 3 and $1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2 > 0$, for the kurtosis to be positive, the restriction should hold.

The same fitting procedure applies to a general GARCH (p, q). The GARCH (1, 1) model describes the time evolution of the average squared errors, that is., the magnitude of uncertainty. Nevertheless, they fail to capture volatility clustering (Bollerslev. 1986).

2.3.1.3 GARCH - M Model

In financial markets, high risk is frequently expected to result in high returns. In such a case, the GARCH-M model postulated by Engle, Lilien, and Robins (1980), where M stands for the mean equation, can be used. The said model extends the traditional GARCH conceptual model by allowing the conditional mean in a series to rely on its conditional variance. A simple illustration is the GARCH-M (1, 1) model expressed as:

$r_t = \mu + \gamma\sigma_t^2 + y_t \quad where \quad y_t = \sigma_t\varepsilon_t \quad for \quad \varepsilon_t \sim N(0, \delta_t^2)$ or

$$\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \beta_1\sigma_{t-1}^2 \quad (2.25)$$

Where parameters μ *and* γ are constants and γ is the risk premium. A positive γ implies that a series is positively relates to its volatility.

Engle's (1982) & Bollerslev's (1986) ARCH and GARCH models cannot distinguish how positive and negative news affects the variance of a series. The EGARCH model, in contrast, has three parameters and allows for an infinite squared roots to affect the present conditional variance. This feature allows the EGARCH model to be somewhat more parsimonious than the ARCH (p, q) models. The EGARCH (p, q) models is an improvement since it includes an overall feature of conditional heteroscedasticity. Some aspects of the GARCH (p, q) models can be enhanced to best reflect the features and dynamics of a specific time series, such as leverage effects, volatility clustering, leptokurtosis, and mesokurtic, common in financial time series.

2.3.2 Asymmetric-GARCH Models

Most financial time series data, such as the asset prices, often react more to "bad" news than "good' news (Omari & Mwita, 2017). Such a phenomenon is so-called leverage effect, fist noted by Black (1976), and is well captured by asymmetric models. Some of the asymmetric models are discussed below.

2.3.2.1 The Exponential GARCH Model

Although the GARCH model adequately captures the heavy tail properties and volatility clustering of financial time series, it is a poor model that cannot capture the leverage effect because conditional variance depends on the modulus of lagged observations (Abdalla, 2012). Volatility in financial time series behaves differently based on the direction of the shock (positive or negative). The asymmetric property is known as leverage effects, and explains how a negative shock increases volatility more than a positive shock of the same size would. The EGARCH model can be used to capture such an asymmetry. It captures the asymmetric innovations of a time-varying variance to shocks while restricting the variance to be positive. Nelson (1991) founded the EGARCH model, which included leverage effects and asymmetry terns. It is described in the conditional variance equation 2.26

$$Ln(\delta_t^2) = \omega + \sum_{j=1}^{p} \beta_j Ln\left(\delta_{t-j}^2\right) + \sum_{i=1}^{q} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\delta_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_i \frac{\varepsilon_{t-i}}{\delta_{t-i}} \tag{2.26}$$

The model is asymmetric since the level $\frac{\varepsilon_{t-i}}{\delta_{t-i}}$ is incorporated with coefficient γ_i. Since the coefficient is negative, positive returns shocks produce low volatility than negative shocks; keeping other factors constant. To capture the asymmetric response of the time-varying variance to shocks, the EGARCH (1, 1) is used alongside the mean equation as illustrated in equation 2.30.

Equation 2.27 is the mean equation:

$$r_t = \mu + \varepsilon_t \tag{2.27}$$

And the Variance equation in 2.28.

$$Ln(\delta_t^2) = \omega + \beta_1 Ln(\delta_{t-1}^2) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\delta_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_1 \frac{\varepsilon_{t-1}}{\delta_{t-1}} \tag{2.28}$$

For EGARCH (p, q) model, the one-step-ahead conditional variance forecast $\hat{\delta}_{t+1/t}$ is estimated using equation 2.29

$$Ln(\delta_t^2) = \omega + \sum_{j=1}^{p} \beta_j Ln\left(\delta_{t-j+1}^2\right) + \sum_{i=1}^{q} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i+1}}{\delta_{t-i+1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_i \frac{\varepsilon_{t-i+1}}{\delta_{t-i+1}} \tag{2.29}$$

2.3.2.2 The Glosten, Jagannathan and Runkle GARCH (GJR-GARCH) Model

The GJR-GARCH model was developed by Glosten, Jagannathan & Runkle (1993). Its variance equation is expressed in equation 2.30.

$$\delta_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \gamma_{t-i} + \sum_{j=1}^{q} \beta_j \delta_{t-j}^2 + \gamma_i I_{t-i} \gamma_{t-i} \tag{2.30}$$

Where α , β, and γ are model coefficients, I is a dummy variable that takes zero (one) when y_{t-i} is positive (negative). If y_{t-i} is positive, negative errors are leveraged (negative shocks or bad news have a higher effect than the positive ones). The parameter of the model is assumed to be positive and that $\frac{\alpha+\beta+\gamma}{2} < 1$. If all leverage coefficients are zero, then the GJR-GARCH model is reduced to a GARCH model. The TGARCH model of Zakoian (1994) is similar to the GJR-GARCH but models the conditional standard deviation instead of the conditional variance. The corresponding one-step ahead conditional variance forecast in the case of the GJR-GARCH (p, q) model is

$$\delta_{t+1/t}^2 = \omega + \sum_{i=1}^{p} \alpha_i \gamma_{t-i+1} + \sum_{j=1}^{q} \beta_j \delta_{t-j+1}^2 + \gamma_i I_{t-i+1} \gamma_{t-i+1} \tag{2.31}$$

2.3.2.3 The Power GARCH (PGARCH) Model

Ding, Engle & Granger (1993) developed the APARCH (p, q) specification, to capture asymmetry in a series. The variance equation of APARCH (p, q) can be expressed as in equation 2.32.

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} (\alpha_i |\gamma_{t-i}| - \gamma_i \gamma_{t-i})^\wedge\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \qquad (2.32)$$

Where; $\omega > 0,\ \delta > 0,\ \alpha_i \geq 0, -1 < \gamma_i < 1,\ i = 1, \ldots, p,\ \beta_j \geq 0,\ j = 1, \ldots, q,\ \ and\ \beta,$ are the standard ARCH and GARCH parameters,γ_i are the leverage parameters, and $\delta$is the parameter for the power term.

The symmetric model sets $\gamma_i = 0,$ for all i. When $\delta = 2$ Equation 2.32 is reduced to a standard GARCH model that captures leverage effect, and when $\delta = 1$the conditional standard deviation was estimated Granger (1993). Additionally, the flexibility of the APARCH model can be enhanced by incorporating δ as an additional parameter. For the APARCH model, one-step-ahead conditional variance forecasting is given by equation 2.33.

$$\sigma_{t+1/t}^\delta = \omega + \sum_{i=1}^{p} (\alpha_i |\gamma_{t-i+1}| - \gamma_{i+1} \gamma_{t-i+1})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j+1}^\delta \qquad (2.33)$$

2.3.2.4 Threshold GARCH Model

The TGARCH model (Glosten *et al*., 1999) is an advancement of EGARCH and the GJR-GARCH model. Given Y_t is the $i.i.d$ random variable with E (Y_t) = 0 and Var (Y_t) = 1. Then (e_t) is the threshold GARCH process (p, q) satisfying an equation 2.34.

$$\varepsilon_t = \sigma_t Y_t$$

$$\sigma_t = \omega + \sum_{i=1}^{p} \omega_i^{(1)} \varepsilon_{t-i}^{(1)} - \omega_i^{(2)} \varepsilon_{t-i}^{(2)} + \sum_{j=1}^{q} \gamma_j\, \sigma_{t-j} \qquad (2.34)$$

Where $\varepsilon_{t-i}^{(1)}$ = max $(e_t$, 0), $\varepsilon_{t-i}^{(2)}$= min $(e_t$,0) dan e_t = $\varepsilon_{t-i}^{(1)}$ - $\varepsilon_{t-i}^{(2)}$ are the effects of the threshold. $i = 1, 2, \ldots \ldots \ldots \ldots p \quad and\ j = 1, 2, \ldots \ldots \ldots q, \gamma_j = random\ variable$

For the TGARCH (p, q) model, the one-step-ahead forecasting of the conditional variance is given by equation 2.35

$$\sigma_{t+1/t} = \omega + \sum_{i=1}^{p} \omega_{i+1}^{(1)} \varepsilon_{t-i+1}^{(1)} - \omega_{i}^{(2)} \varepsilon_{t-i+1}^{(2)} + \sum_{j=1}^{q} \gamma_j \, \sigma_{t-j+1} \qquad (2.35)$$

σ_{t-i}: is the conditional variance and,

ε_t: Disturbance term.

p and q are the orders of the GARCH and ARCH terms, respectively i.e., the number of lagged μ^2 and V^2 terms, respectively.

2.3.2.5 Integrated GARCH Model

The IGARCH models are the unit-root GARCH models. Like the ARIMA models, a vital attribute of IGARCH models is that the effect of past squared shocks $n_{t-i} = a_{t-i}^2 - \sigma_{t-i}^2$ for $i > 0$ on a_t^2 is persistent. The IGARCH assumes that the persistence parameter is equal to 1 hence does capture features such as unconditional variance (Engle & Bollerslev. 1986). An IGARCH (1, 1) modal is expressed in 2.36.

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2 \qquad (2.39)$$

Where ε_t is defined as before and$1 > \beta_1 > 0$.

2.4 Some Applications of GARCH Type Models

Conditional heteroskedastic models have been applied in various econometric and financial time series data. The most common area where the GARCH-type class of models has been commonly applied is the stock market to model and forecast stock returns. Alberg, Shalit, & Yosef (2008) studied estimating stock market volatility using two Tel Aviv Stock Exchange (TASE) indices: TA100 and TA25. The TA25 dataset consisted of 3 058 daily indices from 20th October 1992 to 31st May 2005, whereas TA100 data had 1 911 daily indices from 2nd July 1997 to 31st May 2005. The authors compared symmetric and asymmetric GARCH models. Among the models tested were the standard GARCH and three asymmetric models (EGARCH, GJR-GARCH, and APARCH). The two Student' *t*-distributions outperformed the normal density distribution in both series and across the models. Besides, the asymmetric GARCH models had better forecasting ability than the standard GARCH based on the AIC and log-likelihood values. Among competing asymmetric GARCH models, the EGARCH model using a skewed Student- *t* distribution for TA100 index data (AIC = 2.697; Log-Likelihood = -2566.9) and the TA25 index (AIC = 2.665 and Log-Likelihood = -

4065.4) was the best fit model than the asymmetric GARCH, GJR and APARCH models.

Goudarzi & Ramanarayanan (2011) investigated the impact of good and bad news on volatility in the Indian stock market amidst the 2008-2009 global financial crisis. Two classes of asymmetric volatility models were evaluated (EGARCH and TGARCH) using the Bombay Stock Exchange (BSE) 500 stock index. The daily BSE500 stock index covered ten years from 6th July 2000 to 20th January 2009. The models were estimated using the robust method of Bollerslev-Wooldridge's QMLE assuming a Gaussian distribution. Among the competing orders of EGARCH and TGARCH models, TGARCH (1,1) {AIC = -5.6729, SBIC = -5.6568) was the best fit model for the BSE500 series. The study results showed that the BSE500 returns react to the good and bad news asymmetrically, as a common stylized fact in financial series. The results further supported the presence of leverage effect in the BSE500 returns, indicating that negative news has more impact on volatility than positive news.

Wasiuzzama & Angabini (2011) examined the volatility of the Malaysian stock market using both symmetric and asymmetric GARCH models. The Kuala Lumpur Composite Index (KLCI) was split into two sets' one from June 2000 to the December 2007 (before the 2007/08 financial crisis) and the second from June 2000 to March 2010 (after the crisis). The study findings favoured AR (4) as the best fit model for the conditional mean equation. The GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1) were the best fit for the conditional variance equation. The three conditional heteroscedastic models significantly captured the ARCH effect and volatility clustering in the two time periods. As common features in a financial time series, the KLCI exhibited leptokurtosis, clustering effect, asymmetry, and leverage effect.

Ugurlu, Thalassinos and Muratoglu (2014) modelled volatility in the stock markets for four European and Turkey countries using GARCH models. The authors used daily data from Bulgaria (SOFIX), the Czech Republic (PX), Poland (WIG), Hungary (BUX), and Turkey (XU100). The study results revealed that GARCH, GJRGARCH, and EGARCH effects exist in PX, BUX, WIG, and XU returns. The SOFIX index had no significant GARCH effects. Both markets revealed the presence of volatility clustering of external shocks driven by old news and leverage effect. Besides, the Polish stock market only had the longest memory on variance.

Dana (2016) modelled and estimated volatility in Jordan's stock market using symmetric (ARCH, GARCH) and asymmetric GARCH models (EGARCH). The stocks return volatility for Amman Stock Exchange (ASE) from January 2005 to December 2014. The study results revealed that the two symmetric models capture common stylised facts of financial time series in the ASE: volatility clustering and leptokurtic distribution. The EGARCH model showed evidence of leverage effect in the stock returns at ASE.

Polodoo (2011) examined the effect of exchange rate volatility on economic performance in small island developing countries. The study employed annual data from 1999 to 2010 and computed z-scores as proxy for the exchange rate volatility. Panel ordinary least square regression analysis was employed with heteroscedastic-robust standard errors to correct heteroscedasticity. The study findings showed that exchange rate volatility positively influences economic growth. The study depicts a methodological gap in measuring exchange rate volatility using GARCH class of models.

In Iran, Pahlavani & Roshan (2015) compared ARIMA and hybrid ARIMA-GARCH models (ARIMA-GARCH, ARIMA-IGARCH, ARIMA-GJR, and ARIMA-EGARCH) in forecasting the exchange rate of Iran. The dataset used was the daily exchange rate against the U.S. Dollar (IRR/USD) from 20th March 2014 to 20th June 2015. Based on the minimisation of prediction errors, the study findings indicated that ARIMA ((7,2), (12)) –EGARCH (2,1) was the best fit for the data (RMSE = 0.0007, MAE = 0.0006, and TIC = 0.500069). The model efficiently captured the volatility and leverage effect in the exchange rate returns and produced better forecasts than other models.

The application of asymmetric GARCH models is not limited to financial time series models. Ali (2013) evaluated EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH, and APARCH models to fit functional associations of the pathogen indicators for recreational activities in Huntington Beach, Ohio, United

States. The data used spanned 2006 to 2008, with 225 daily observations. The study compared GED, Student's t, exponential, normal, and inverse Gaussian distributions besides their skewedness. The study findings indicated that TGARCH better fits the pathogens data with turbidity, rainfall, dew point, river flow, and cloud cover being significant predictors (AIC = 1.146, BIC = 1.313, HQ = 1.214 Log Likelihood = -117.940).

Zhang, Yao, He, & Ripple (2019) compared the performance of two regime-switching (MMGARCH and MRS-GARCH) and the single-regime GARCH models (GARCH, GJR-GARCH, and EGARCH) in examining and forecasted the volatility of crude oil market {West Texas Intermediate (WTI) and Brent}. The study findings indicated that the MRS-GARCH model accurately estimated weekly data based on the in-sample forecasts. The out-sample forecasts showed the limited significance of under the regime-switching approach. The authors established no significant difference between the regime-switching and the single-regime GARCH models.

Atoi (2014) modelled the volatility of Nigeria's All Share Index from January 2, 2008, to February 11, 2013, using the first-order symmetric and asymmetric GARCH models with an assumption of Normal, Student's-t, and generalized error distributions. The study findings established the leverage effect, indicating that volatility responds more to bad than the good news of the same threshold. The best fit volatility model based on the minimisation of RMSE and Theil Inequality Coefficient was the Power-GARCH (1, 1) with student's t error distribution. The study recommended that future empirical works consider alternative error distributions to obtain a robust volatility forecasts that guarantees a sound policy decision.

Oberholzer & Venter (2015) compare how GARCH (1,1), GJR-GARCH (1,1), and EGARCH (1,1) models to analyse the 2007-2009 financial crisis caused volatility in the five indices on the Johannesburg Stock Exchange (JSE). The authors relied on 3, 326 daily closing prices for the top forty indexes (J200), Mid Cap Index (J201), Small Cap Index (J202), All-Share Index (J203) and Fledgling Index (J204) spanning January 2002 to end February 2014. Based on the minimization of AIC and SIC, the GJR-GARCH (1,1) best fitted all the indices except the J204, where the EGARCH (1,1) model was the best fit. Moreover, the results indicated the presence of leverage effect in all the series.

Bouseba & Zeghdoudi (2015) centred their study on GARCH models to examine VaR of monthly oil price data from January 1, 2009 to December 31, 2014, consisting of 2, 192 data points. The authors discovered that normal GARCH models explain the non-normal distribution of energy prices. In light of this, the error term will, as a result, exhibit Skewness and leptokurtic distribution. Normal GARCH VaR estimates perform better than the usually employed by energy companies. He recommended using the stable GARCH for accounting for the non-Gaussian distribution of the energy returns and volatility.

The ARCH-type class of models has also been used to model inflation. Benedict (2013) evaluated how the ARCH-type models can capture the volatility of Ghana's monthly inflation from January 1965 to December 2012. The author compared three models: ARCH, GARCH, and the EGARCH. Overall, the study findings demonstrated that the EGARCH (2, 1) best fitted the data outperforming other models in terms of minimisation of AIC (5.09), BIC (5.16), and MAE (2.88). As opposed to the common stylized fact, the inflation data showed an absence of the leverage effect since positive shocks increased the volatility of the inflation rate more than the negative shocks of equal size.

Ngailo, Luvanda, & Massawe (2014) focused on modelling Tanzania's inflation rate using ARCH family models. The study used monthly data from January 1997 to December 2010, constituting 168 observations. Based on the minimization of the prediction errors, evaluation metrics showed that the GARCH (1, 1) model best fitted the data (AIC = 474.8, BIC = 487.3, log-likelihood = 233.4). In Ghana, Nortey et al. (2014) compared the ARCH, GARCH, and EGARCH models using Ghana's monthly inflation from January 2000 to December 2013. The study findings indicated that the EGARCH (1, 2) model with $ARIMA\ (3, 1, 2)\ \times\ (0, 0, 0)_{12}$ as the mean equation best fitted the data. The mean equation outperformed other competing ARIMA type models with AIC and the BIC values of 3.41 and 3.58, respectively. The variables equation for the mean model residuals has AIC and BIC values of 2.49 and 2.76, respectively, as the least scores compared to other competing ARCH, GARCH, and EGARCH models. The study has illustrated that asymmetric models (in this case, the EGARCH model) outperform the standard ARCH and GARCH models. Thus, in the current study, we compare competing asymmetric models only as stated previously.

Onwukwe, Bassey, & Isaac (2011) compared three heteroscedastic models, namely: GARCH (1,1), EGARCH (1,1), and GJR-GARCH using time series behaviour of daily stock returns of four firms (UBA, Unilever, Guinness, and Mobil) listed in the Nigerian over the period between January 2, 2002 and December 31, 2006. As stylized features of financial series, the return series of each firm exhibited leverage effect, leptokurtosis, volatility clustering, and negative skewness. The study findings indicated that the GJR-GARCH (1, 1) produces a better fit to all the return series in both the in-sample and out-of-sample forecasts evaluation period. The RMSE for the model of UBA, Mobil, Unilever, and Guinness returns were 1.326, 1.087, 1.639, 1.308, respectively. In extension, the current study compared the five asymmetric conditional heteroskedasticity classes of models by including TGARCH and PGARCH using Kenya's monthly exchange rate (January 1993 to June 2021) and BoP data (August 1998 to June 2021).

GARCH-type class of models has also been applied to exchange rate data. Thorlie, Song, Wang, & Amin (2014) modelled the Sierra Leones' exchange rate volatility using asymmetric GARCH models. The ARMA, GARCH, and asymmetric GARCH models were compared using the Leones/USA dollars exchange rate returns computed from the monthly data from January 2004 to December 2013. The study findings demonstrated that the Asymmetric (GARCH) and GARCH model assuming non-normal than the normal distribution better estimated the conditional variance in the series. Based on AIC and BIC, the GJR-GARCH model using the skewed Student t- distribution was the best fit for the Sierra Leone exchange rate volatility (AIC -7.5929, BIC = -7.4061)). Leverage effect and asymmetry were also present in the exchange rate returns as a common stylized fact of financial time series.

Kandora (2016) modelled exchange rate volatility using asymmetric GARCH Models in Sudan. A comparison of ARMA, GARCH, and Asymmetric GARCH models was done using the monthly Sudanese Pound SDG/USA dollars exchange rate return series from January 1999 to December 2013. The study employed the AIC and BIC to select the best fit model among several competing mean and variance models. The best fit mean model was ARIMA (1, 1, 2) with AIC and BSC values of -3.7944 and -3.7229, respectively. Overall, the conditional variance models were better when fitted assuming the student t-distribution than normal distribution. The inclusion of the variance equation showed that ARIMA (1,1,2) - DGE-GARCH (1, 1) {AIC = -7.844, BIC = -7.665} outperformed ARIMA (1,1,2) - GARCH (1,1) (AIC = -7.501, BIC = -7. 357), and ARIMA (1,1,2) - GJR-GARCH (1, 1) {AIC = -7.709, BIC = -7.547}. Evaluation of the model parameters showed the existence of leverage effect in the series. The authors also demonstrated that the asymmetric GARCH models show asymmetry in exchange rate returns. As a comparative study, the current study evaluates whether Kenya's exchange rate shows a leverage effect and examines the nature of its volatility using a selected asymmetric ARCH type class. The analysis is also extended to the balance of payment series.

The study of Nwoye (2017) examined the volatility of the exchange rate of Nigeria Naira against the US Dollar using GARCH family models (GARCH, EGARCH, GJR –GARCH, AVGARCH, TGARCH, NGARCH, NAGARCH, APARCH, ALLGARCH, and GARCH). The author used monthly exchange rates data from January 1999 to December 2012. The EGARCH (2,2) best fitted the data (AIC = -6.497, BIC = -6.339, HQ = -6.433). The model demonstrated evidence of volatility clustering, leverage effect, and asymmetric effect in the naira exchange rate against the USD.

Maqsood *et al*. (2017) employed the GARCH type class of models to examine the volatility of the daily returns of the Kenyan stock market. The returns were computed using the daily closing prices of the Nairobi Securities Exchange (NSE) index from March 18, 2013 to February 18, 2016 (730 data points excluding public holidays). The study compared both symmetric and asymmetric type, heteroscedastic models. Based on the AIC and BIC minimization criterion, TGRACH (1,1) {AIC = -8.827, BIC -8.795} best capture the volatility clustering and leverage effect outperforming other heteroscedastic models; which are: GARCH (1,1), GARCH-M (1,1), EGARCH (1,1) and PGARCH (1,1). In extension, the current study compared the asymmetric

Conditional Heteroskedasticity class of models to include TGARCH and GJR-GARCH, which was not included in the study. Besides, the exchange rate and BoP data spanned from January 1993 to June 2021 and from August 1998 to June 2021, respectively.

Petrică & Stancu (2017) empirically examined how symmetric (ARCH and GARCH) and the asymmetric GARCH models (EGARCH, TARCH, and PARCH) can capture the volatility of daily returns of EUR/RON exchange rate. The data set spanned 4th January 1999 to 13th June 2016, constituting 4439 observations. The model parameters were estimated using the MLE method assuming several distributions: Normal, Student's t, GED, Student's with fixed degrees of freedom, and GED with fixed parameters. The best model for data was EGARCH (2,1), assuming that the data follows a student's t distribution (AIC $\approx$ 0.7880). Recently, Aliyev, Ajayi & Gasim (2020) evaluated asymmetric market volatility using EGARCH and GJR-GARCH. The analysis relied on daily data Nasdaq-100 series from 4th January, 2000, to 19th March, 2019. The study revealed that the volatility and leverage effect were present. Besides, Nasdaq-100 index returns' volatility exhibited clustering, a good attribute for investors who hedge against low and high prices.

In Kenyan, Okeyo, Ivivi, & Ngare (2016) modelled inflation volatility using the ARCH -type class of model. The authors used inflation data from January 1985 to April 2016. Three ARCH family type models (ARCH, GARCH, GJR GARCH, and the EGARCH) were compared. The study findings indicated that the EGARCH (1, 1) with GED was the best model to forecast Kenya's inflation. The study recommended that policy makers involved in forecasting inflation rates ponder Heteroscedastic models because they capture volatilities.

In addition, Fwaga *et al* (2017) evaluated an effective Arch-type class of model for forecasting Kenya's monthly inflation. The data spanned January 1990 to December 2015. The study compared competing orders of the standard GARCH models EGARCH (1, 1). The ARCH effects test using the Engle Arch test showed that heteroscedasticity is present in the inflation return series. The study evaluated the competing models using AIC and BIC values. The results showed that EGARCH (1,1)

{AIC = -0.1668; BIC =-0.10698} model was best fit model for forecasting Kenyan inflation data outperforming GARCH (1,1), GARCH (1,2), GARCH (2, 1) and GARCH (2,2). The current study has important findings relevant to the current study. The study has demonstrated that the EGARCH model, an asymmetric model, outperforms the standard GARCH model. Thus, in the current study, we compare competing asymmetric models only as stated previously.

In Bangladesh, Alam & Rahman (2012) modelled the volatility of the daily foreign exchange rate (BDT/USD) using GARCH type models (which included GARCH, EGARCH, TARCH, and PARCH). The performance of the selected GARCH class of models was compared with the AR and ARMA models. The data used spanned 3rd July 2006 to 30th April 2012, making up 1513 trading days. The crucial finding from the study is that the GARCH type models demonstrate the existence of volatility cluttering and leverage effect. The EGARCH and TARCH models, been asymmetric ARCH models, outperformed all the other competing models in fitting annualized returns and transaction cost in both in and out-sample data set. Therefore, the current study compared asymmetric models with the inclusion of GRJ-GARCH and APARCH while fitting and forecasting the volatility of Kenya's exchange rate data and BOP.

In Kenya, Omari, Mwita, & Waititu (2017) modelled Kenya's exchange rate in US dollars to Kenya Shilling (USD/KES) volatility using GARCH models. The data used was daily rates spanning January 3, 2003 to December 31, 2015, constituting 2818 observations. The authors employed symmetric (GARCH (1, 1) and GARCH-M (1,1,)) and asymmetric models (EGARCH (1, 1), GJR-GARCH (1, 1), and APARCH (1, 1)). Based on the minimization of BIC and AIC, the ARMA (2, 0) was the best fit mean equation (AIC = 213, BIC = 213). Overall, the asymmetric models (EGARCH (1, 1) (AIC = -8.2004, BIC = 8.1857), GJR-GARCH (1, 1) (AIC = -8.5177, BIC = -8.5029) and APARCH (1, 1)) (AIC = -8.5119, BIC = -8.4950)) outperformed the symmetric models (GARCH (1, 1) (AIC = -8.5166, BIC = -8.5039), and GARCH-M (1,1)) (AIC = -8.8778, BIC = -8.8630). The symmetric models showed the existence of asymmetry and leverage effect in the series ($\gamma < 0$). Due to the better performance of asymmetric than the symmetric models, the authors suggested that future studies can include other asymmetric GARCH-type models. Thus, the current study includes GJR-GARCH and

TGARCH for comparison using Kenya's exchange rate data from January 1993 to June 2021.

2.5 Summary of ARCH-type Models

Table 1: Summary of ARCH type class of Models.

Model	Type of model	Leverage effect property	Ability to embrace thick tail property
ARCH	Symmetric	Fail to capture leverage effects	Does not fully embrace the thick tail
GARCH	Symmetric	Fail to capture leverage effects	Does not fully embrace the thick tail
M-GARCH	Symmetric	Fail to capture leverage effects	Does not fully embrace the thick tail
EGARCH	Asymmetric	Captures the leverage effect by capturing the asymmetric innovations	Captures thick tail property.
GJR-GARCH	Asymmetric	Captures leverage effect property	Captures thick tail property.
TGARCH	Asymmetric	Captures leverage effect property	Captures thick tail property.
PGARCH	Asymmetric	Captures leverage effect property	Captures thick tail property.
IGARCH	Asymmetric	Captures leverage effect property	Captures thick tail property.

CHAPTER THREE
METHODOLOGY

3.1 Location of the Study

This study focuses on Kenya's Economy in examining the applicability of conditional heteroscedastic class of models, namely; Asymmetric-GARCH type models, to fit and predict exchange rate volatility and BoP in Kenya.

3.2 Research Design

The study used descriptive research design. The descriptive research design explores the stylized fact properties of the two time series data including volatility clustering, negative kurtosis, and excess skewness. The features were determined by the use of data visualization and descriptive statistics. Besides, the two variables are time series data. The selected Asymmetric-Arch type models learn the past behaviour of the series to determine the best-fit parameters for the data and forecast future occurrences from the best fit model. Thus, the study is partly retro-prospective (looking into the past) and prospective (looking into the future).

3.3 Data Collection

The current study employed time series analysis hence no defined population and sample size. The analysis used monthly exchange rates data in Kenya from January 1993 to June 2021 as a convenience sample available during the study. The research used secondary data obtained from the Central Bank of Kenya (CBK) website (https://www.centralbank.go.ke/rates/forex-exchange-rates/). The Kenya's exchange rate is referenced to the US dollar. The period is suitable since the sample covers the period when Kenya was already on a flexible exchange rates regime. The secondary data on BoPs was also extracted from the CBK website (https://www.centralbank .go.ke/statistics/balance-of-payment-statistics/). The monthly BoP data spanning from August 1998 to June 2021. The respective series was downloaded in October 2021, and have 342 data points and 275 data points considered adequate for a time series analysis technique.

3.4 Data Analysis

The R statistical software (R Core Team, 2016) was used to analyse the data. Preliminary analysis done included descriptive statistics, trend analysis, and stationarity test. The selected asymmetric GARCH models (IGARCH, TGARCH, APARCH, GJR-GARCH, and EGARCH) were fitted to the stationary log-differenced data based on the functions in the RUGARCH (Ghalanos, 2022).

3.4.1 Fitting the Asymmetric-GARCH Type Models

The conditional variance of return series is expressed as a function of constant, past volatility, and past forecast variance in the generalised ARCH model (Engle, 1982 & Bollerslev, 1986). The model parameters were estimated using the MLE method.

3.4.1.1 The GARCH Model

The conditional variance for the generalised ARCH (p, q) model is defined as:

$$V_t^2 = \beta_0 + \sum_{r=1}^{K} \alpha_r \, \mu_{t-r}^2 + \sum_{v=1}^{S} \beta_v \, V_{t-v}^2 \quad (3.1)$$

where$\beta_0 > 0,\ \alpha_i \geq 0, \beta_j > 0$

V^2: is the restricted variance

μ^2: Error term referred to as disturbance term.

K is the order of the generalised terms, i.e., the amount of lagged μ^2 terms

S is the size of the ARCH terms, i.e., the amount of lagged V^2 terms,

Both β_v and α_r are greater than zero, and the component $\sum_{r=1}^{K} \alpha_r + \sum_{v=1}^{S} \beta_v < 1$ to achieve stationary. Additionally, the restraints $\alpha_r \geq 0$ and $\beta_v \geq 0$ ensures that $V^2$is strictly positive (Poon, 2005).

3.4.1.2 Exponential GARCH (EGARCH) Model

The EGARCH model with its leverage and asymmetry properties in its equation is defined with conditional variance written as:

$$Ln(V_t^2) = \omega + \sum_{v=1}^{S} \beta_v Ln\,(V_{t-v}^2) + \sum_{r=1}^{K} \alpha_r \left\{ \left| \frac{\mu_{t-r}}{V_{t-r}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma_r \frac{\mu_{t-r}}{V_{t-r}} \quad (3.2)$$

$V^2 =$ the conditional variance and,

$\mu^2 =$ Disturbance error term.

K = the order of amount of lagged μ^2 terms

S= the order of the number of lagged V^2 terms.

The model is asymmetric in nature because the component $\frac{\mu_{t-r}}{V_{t-r}}$ is included with coefficient γ_r. Since the constant is negative, positive returns shockwaves cause less volatility than negative return shocks when other factors remain constant.

3.4.1.3 Glosten, Jagannathan and Runkle GARCH(GJR-GARCH) Model

The GJR-GARCH variance equation is defined by;

$$V_t^2 = \omega + \sum_{r=1}^{K} \alpha_r \gamma_{t-r} + \sum_{v=1}^{S} \beta_v \delta_{t-v}^2 + \gamma_r I_{t-r}\gamma_{t-r} \quad (3.3)$$

Where $\alpha, \beta, and \ \gamma$ are model parameters.

K and S are the lagged orders of the γ and δ_{t-v}^2 terms,

$I =$ Is a dummy variable, also known as the indicator function, and takes the value zero when the parameter γ_{t-r} is negative and one if it is positive. If γ is positive, negative shocks have a larger effect size than positive shocks. The model parameters are assumed to be positive and that $\frac{\alpha+\beta+\gamma}{2} < 1$, if leverage coefficients are zero, then the GJR-GARCH model becomes the GARCH model.

3.4.1.4 Power GARCH (PGARCH)Model

The PGARCH or APARCH (K, S) has the variance equation written as

$$V_t^{\delta} = \omega + \sum_{r=1}^{K} (\alpha_r |\gamma_{t-r}| - \gamma_r \gamma_{t-r})^{\wedge}\delta + \sum_{v=1}^{S} \beta_v V_{t-v}^{\delta} \quad (3.4)$$

Where;

$\omega > 0, \delta > 0, \alpha_r \geq 0, -1 < \gamma_r < 1, r = 1, \dots, K, \ \beta_v \geq 0, v = 1, \dots, S, \ \alpha \ and \ \beta,$

, are the normal ARCH and generalised ARCH parameters, γ_i is the leverage effect parameter and δ is the power term.

3.4.1.5 Threshold GARCH (TGARCH) Model

The TGARCH model is an extension of the exponential GARCH and the GJR-GARCH model. Its conditional variance is given by

$$V_t = \omega + \sum_{r=1}^{K} \alpha_r^{(1)} \mu_{t-r}^{(1)} - \alpha_r^{(2)} \mu_{t-r}^{(2)} + \sum_{v=1}^{S} \gamma_v V_{t-v} \quad (3.5)$$

Where $\mu_{t-r}^{(1)}$ = max (e_t, 0), $\mu_{t-r}^{(2)}$= min (e_t, 0) dan e_t = $\mu_{t-r}^{(1)}$ - $\mu_{t-r}^{(2)}$ are the effects of the threshold.

K and S are the lagged orders

3.4.1.6 Integrated GARCH (IGARCH) Model

The IGARCH model is similar to the ARMA model and is a unit-root GARCH model. A key feature of the IGARCH model is that the effect of previous squared residuals $\pi_{t-r} = a_{t-r}^2 - V_{t-r}^2$ for r >0 on a_t^2 is persistent. A variance IGARCH (K, S) model is expressed in equation 3.6.

$$V_t^2 = \omega + \sum_{r=1}^{K} \beta_{\mathrm{r}} V_{t-\mathrm{r}}^2 + \sum_{v=1}^{S} (1-\beta_{\mathrm{r}})\, a_{t-\mathrm{v}}^2 \qquad (3.6)$$

Where $1 > \beta_r > 0$. V^2 = the conditional variance

K and S are the order of the number of lagged $V_{t-\mathrm{r}}^2$, and lagged $a_{t-\mathrm{v}}^2$ terms, respectively.

3.4.2 Estimation of Model Parameters

The MLE method was used in parameter estimation. Basically, the method works by looking for the optimum parameter values for the given data (Brooks, 2008). In the form of restricted heteroscedasticity, the mean and variance GARCH (1, 1) is defined as;

$$X_t = \mu + \varepsilon_t \qquad (3.7)$$

Where $\varepsilon_t \sim N(0, \delta_t^2)$

$$\delta_t^2 = \beta_0 + \alpha_1 a_{t-1}^2 + \beta_1 \delta_{t-1}^2$$

Where the variance of the errors terms δ_t^2 is time-varying. The log-likelihood function (LLF) by Weiss, Bollerslev, and Wooldridge (1986) and Brooks (2008) for the disturbances is defined as;

$$L = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{T} \log \delta_t^2 - \frac{1}{2}\sum_{i=1}^{T} \frac{(X_t - \mu - \emptyset r_{t-1})^{\wedge}2}{\delta_t^2} \qquad (3.8)$$

where the component $-\frac{1}{2}\log(2\pi)$is a constant which depends on the parameters. T is the number of observations and r_t is exchange rate return.

Maximization of the log-likelihood function (LLF) necessitates the minimization of $\sum_{i=1}^{T} \log(\delta_t^2)$, $\sum_{i=1}^{T} (X_t - \mu - \emptyset X_{t-1})^2/(\delta_t^2)$ and the variance error term (Brooks, 2008). The standard normal distribution does not capture fat tails in a series. Other distributions such as the GED or student distribution are often used. The MLE of the parameters is achieved by statistical maximization of the LLF using the Marquardt algorithm (Bollerslev & Wooldridge, 1992).

3.4.3 Model Selection Criteria

The model selection criteria examine whether the fitted model optimally balances the goodness-of-fit and parsimony. Several evaluation criteria have been adopted to assess the model performance of competing models or orders. Some common criterion is the maximum likelihood ratio test of the models, where a model with the highest log-likelihood value is the best (Shephard, 1996). Suppose the competing models do not have equal parameters. In that case, the principle of parsimony applies, such that the best model minimises criterion such as the AIC, BIC, Schwarz information criterion (SIC), and the Hannan–Quinn (HQ). The best fit model was determined based on parsimony (AIC, BIC, Log-Likelihood criterion) and minimisation of prediction production errors (ME and RMAE). The five metrics are estimated using equations 3.9 to 3.11 (Zhang Haonan, 2013)

$$AIC = -2\log(L) + 2\log(p+q) \tag{3.9}$$

Where L indicates the likelihood of the data with a certain model, p and q indicate the lagged orders of AR and MA terms, respectively.

$$BIC = -2log\,(L) + \;2(m) \tag{3.10}$$

Where n and m are the numbers of observations and parameters in the model, correspondingly, and log (L) is the log-likelihood. The best model is the one that minimizes the AIC or BIC while maximizing the log-likelihood.

3.4.4 Model Evaluation Criterion

In-sample forecasting ability helps determine the best model to be adopted (Clement, 2005). These approaches of sample model selection criteria for assessing the predictive ability of competing models include the Root Mean Square Error (RMSE) and Theil Inequality Coefficient (TIC), Mean Squared Error (MSE), Mean Absolute Error (MAE), and Adjusted Mean Absolute Percentage Error (AMAPE). If ${\delta^2}_t$ and $\widehat{\delta^2}_t$ represents the actual and forecasted volatility/variance of a series at time t, then; MSE, measuring the average of the squared individual errors, is estimated using equation 3.11 (Poon and Granger, 2003);

$$MSE = \frac{1}{h+1}\sum_{t=s}^{s+h} \widehat{\delta^2}_t - \delta^2{}_t \tag{3.11}$$

Where; h is the number of head steps, and s is the sample size.

A model that minimises the MSE value is a better fit for a given data. RMSE is simply obtained by taking the square root of MSE and is estimated using equation 3.12:

$$RMSE = \sqrt{\frac{1}{h+1}\sum_{t=s}^{s+h} \widehat{\delta^2}_t - \delta^2{}_t} \tag{3.12}$$

The best asymmetric GARCH model is the one that minimizes the MAE, and RMSE

3.4.5 Residual Diagnostics

An adequate model for forecasting or, in this case, evaluation of volatility should have residuals similar to a series generated from white noise. For simplicity, this can be examined by visualization of the residuals in a time plot. A histogram superimposed with a density plot was also used to test if the normality assumption holds. For normally distributed residuals, the density curve should be bell-shaped. In addition, the residuals of a given best model must not be autocorrelated. The Ljung-Box (Q) statistic was employed to examine the presence of autocorrelation. It tests the null hypothesis that there is no serial correlation. The insignificant Q statistic implies absence of serial correlation; hence the model fits the data well.

Both symmetric and asymmetric curves differ by the leverage effect. Let ε_t be a measure of shocks; where a positive value of ε_t depicts a positive shock, and vice versa. A standard GARCH model (symmetric) will have a news impact curve that is quadratic, that is, symmetric and centred around $\varepsilon_{t-1} = 0$ (that is; when there is no bad or good news). In that case, positive and non-positive shocks of the same magnitude yield a similar magnitude of volatility. But conventionally, non-positive shocks can cause higher volatility than positive shocks of the same magnitude. Thus, the GARCH model underestimates the degree of volatility arising from large negative or bad news/shocks.

In the same logic, it overestimates the volatility arising from small positive shocks or good news (Engle & Ng, 1993). To examine the tendencies, three diagnostic tests for volatility models: The Sign Bias Test (SBT), the Negative Size Bias Test (NSBT), and the Positive Size Bias Test (PSBT) are usually carried out. All three tests converge to the evaluation of the model misspecification.

However, as Engle & Ng (1993) described, the bias tests have different methodologies. The SBT augments an indicator function S_{t-1}^{-} which assumes a rate of one if there is negative news ($\varepsilon_{t-1} < 0$) or else zero. It examines whether positive and non-positive shocks affect volatility differently from the fitted model. The NSBT employs a dummy $S_{t-1}^{-}\varepsilon_{t-1}$. It examines if large negative shocks correlate with the volatility contrary to the fitted volatility model projection. On the contrary, the PSBT uses the dummy variable $S_{t-1}^{+}\varepsilon_{t-1}$; where $S_{t-1}^{+} = 1 - S_{t-1}^{-}\varepsilon_{t-1}$. Unlike the NBST, the PSBT examines how larger positive shocks can impact volatility differently from the forecast of the fitted conditional heteroscedastic model.

Let v_t be the normalized residuals at time t of the fitted volatility model $v_t = \frac{e_t}{\sqrt{h_o}}$. The LM test statistic for $H_0\colon \delta_a = 0$ in any given asymmetric model entails testing the null hypothesis; of $H_0\colon \delta_a = 0$ in the auxiliary regression equation 3.13.

$$v_t^2 = \underline{Z}_{0t}\underline{\delta}_0 + \underline{Z}_{at}\underline{\delta}_a + \mu_t \qquad (3.13)$$

where; $\underline{\delta}_0$ is the $k \times 1$ direction of parameters of the null hypothesis; $\underline{Z}_{0t}$ is the k x 1 vector of regressors under the null hypothesis; and $\underline{Z}_{at}$ is the $m\,x\,1$ direction of regressors not included in the model, with associated parameters, $\underline{\delta}_a$, and μ_t is the model residuals. For a perfect or adequate model, the predictors in equation 3.9 should be significant. Thus, the conditional heteroscedastic model is misspecified if the fitted model predicts the squared standardised residual. The model residuals should show no ARCH effects; otherwise, they must be modelled. The Lagrange Multiplier (LM) test was used to test for the existence of ARCH, as was used in the study on each of the best fit models for both exchange rate and BOP. The LM computed using equation 3.14:

$$\xi_{LM} = T \times R^2 \qquad (3.14)$$

where; R^2 is square of the multiple correlation coefficient of equation 3.14, and T is the sample size. The LM test statistic takes an asymptotic χ^2 distribution with m as the degrees of freedom, with m denoting the number of restricted parameters. In either case, the null hypothesis is that; there are no ARCH effects. Significant test results ($p < 0.05$) support the rejection of the null hypothesis and vice versa.

The regression equations used for evaluation the SBT, the NSBT, and the PSBT are presented in equation 3.15.1, 3.15.2, and 3.15.3, respectively.

$$v_t^2 = \alpha + \beta\, S_{t-1}^- + \underline{\beta}'\, Z_{ot}^* + e_t \quad (3.15.1)$$

$$v_t^2 = \alpha + \beta\, S_{t-1}^- \varepsilon_{t-1} + \underline{\beta}'\, Z_{ot}^* + e_t \quad (3.15.2)$$

$$v_t^2 = \alpha + \beta\, S_{t-1}^+ \varepsilon_{t-1} + \underline{\beta}'\, Z_{ot}^* + e_t \quad (3.15.3)$$

Where α and β are parameters of the model, $\underline{\beta}'$ is a vector of parameters associated with the regressors not included in the model, and e_t is a vector of model residuals. The three tests are evaluated based on the t-statistic associated with the coefficient b in their respective equations in 3.11. Alternatively, the three tests can be done jointly following the regression in equation 3.12

$$v_t^2 = \alpha + \beta_1 S_{t-1}^- + \beta_2 S_{t-1}^- \varepsilon_{t-1} + \beta_3\, S_{t-1}^+ \varepsilon_{t-1} + \underline{\beta}'\, Z_{ot}^* + e_t \quad (3.16)$$

The t-statistics proportions for β_1, β_2, and β_3 correspondingly, sign bias, negative, and the positive bias test statistics. It is an expectation that, if adequate volatility, then $\beta_1 = \beta_1 = \beta_1 = 0$, $\underline{\beta}' = \underline{0}$; and hence e_t is $i.i.d.$ If coefficients are significant, the positive or negative external shocks affect the variance differently from the model's predictions. On the contrary, there is no bias if the coefficients are insignificant.

3.4.6 Forecasting

The ultimate goal of time series modelling techniques is to make future forecasts. The model that minimises BIC and AIC values was the best fit and used in the forecasting. After model selection, a one-step-ahead model forecasting will be done by assuming the model parameters are well-known and the innovations have a Gaussian distribution. Both the in-sample predictions (observed) and out-sample predictions (unobserved) were estimated alongside the one standard deviation confidence band where the actual values are likely to lie. 12-step ahead forecasts from July 2021 to June 2022 were made for each variable.

3.5 Ethical Considerations

The Chuka University Ethics Committee approved the research proposal (Appendix I). The research permit was then obtained from NACOSTI before proceeding with the research (Appendix II). Where other peoples' work was used, they were acknowledged through citations to avoid plagiarism.

CHAPTER FOUR
RESULTS AND DISCUSSIONS

4.1 Trend Analysis of Exchange Rate and BoP Data

The study aims to apply asymmetric-GARCH models to exchange rate and BoP in Kenya using a conditional heteroscedastic class of models. Figure 1 (a) shows Kenya's exchange rate data from January 1993 to June 2021. The figure shows an overall increasing trend with notable undulation over time. For instance, there was a sharp decline around 1992, 2007, and 2012, which can be associated with the usual electioneering period associated with election violence which decreases investor confidence, especially by the foreigner. The long periods of exchange rate stability are seen when there is a smooth government transition. For instance, the periods 2003-2005; 2017- 2019 had a relatively stable exchange rate. However, the recent coronavirus disease 2019 (COVID-19) has seen a sharp appreciation in Kenya’s exchange rates.

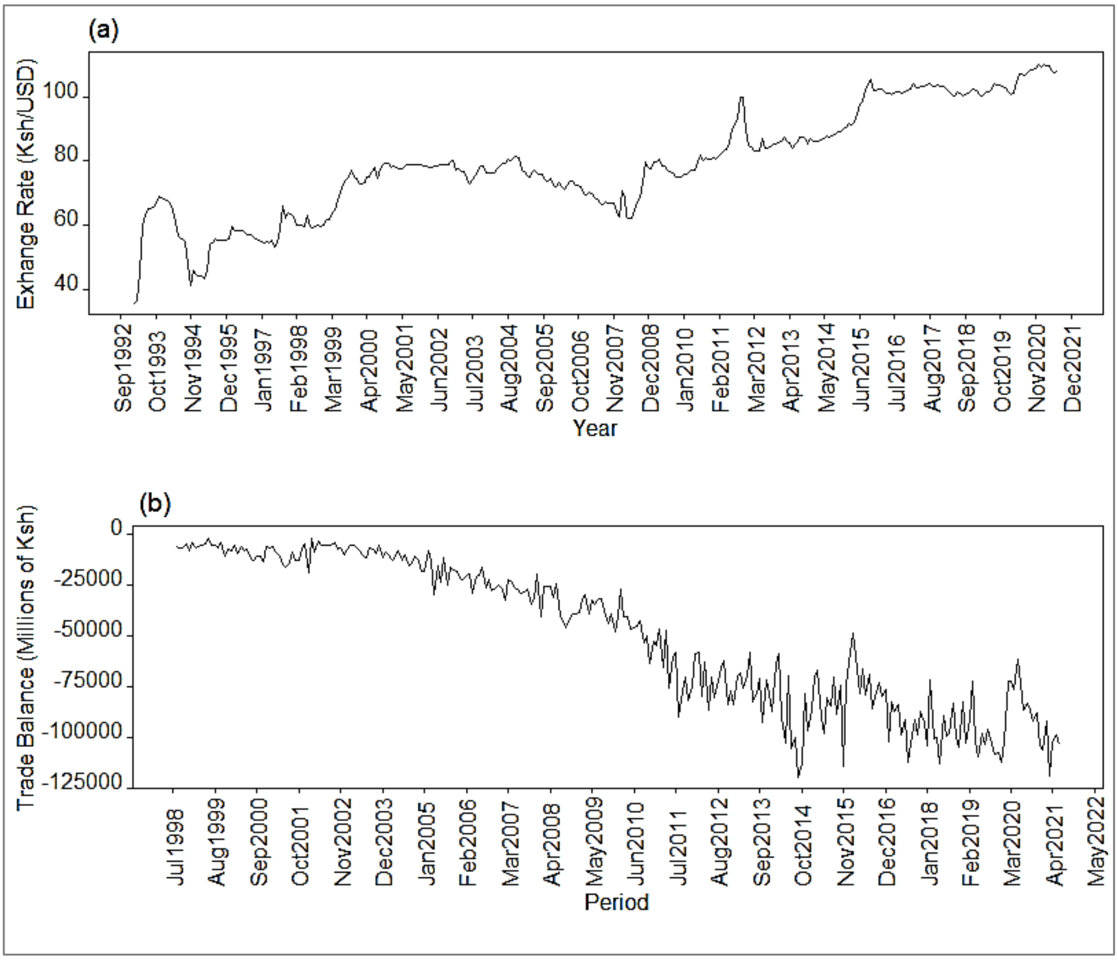

Figure 1: Time plot of Kenya’s exchange rate and BoP (current account) data

The BoPs is a statistical summary statement report showing transactions among citizens and non-residents during a given time period. Figure 1 (b) shows Kenya's monthly current account balance in millions of Ksh from August 1998 to June 2021. Overall, Kenya's indebtedness seems to widen over time. The counterbalancing of merchandise trade (transactions of goods and services), financial service sector and gross primary salary (from investments), and net secondary income (from remittances from Kenyans in diaspora) is vital in the BoP. However, the big deficit has been attributed to the Merchandise trade, where imports have a higher value than the exported raw materials. The key exports are primarily agricultural products, including tea, horticulture, apparel and clothing accessories, coffee, and Tobacco. On the contrary, Kenya's import basket contains high valued finished or intermediate products comprising industrial machinery, petroleum products, iron and steel, road motor vehicles, and medicinal or pharmaceutical products.

There was a sharp decreasing trend from 2003 to 2014. The least BoP recorded in 2014 can be accredited to the decline in foreign exchange incomes from coffee, tea, and horticultural commodities. Later in 2016, the BoP slightly rebounded following the increased net financial inflows partly accruing from the disbursements to support infrastructure projects mainly to the Standard Gauge Railway development project. The Kenya National Bureau of Statistics (KNBS) statistical release 2016, indicated that the public and publicly guaranteed external debt increased by about 10.4 % from about KSh 1.550 trillion as of September 2015 to KSh 1.711 trillion in September 2016. In 2017, there was another sharp decline in the current accounts' BoPs. The total stock of public and publicly guaranteed external debt grew from Ksh. 1.855 trillion in September 2016 to KSh 2.310 trillion as of September 2017. The growth can be attributed to the outstanding syndicated loans disbursed in the first and second quarters of 2017 (KNBS statistical release, 2017). Despite an increase in the earnings from domestic exports by about 6.3% from KSh 122.4 billion in the third quarter of 2016 (2016Q3) to KSh 130.2 billion in the third quarter of 2017 (2017Q3), owing to the increased value of tea exports from KSh 29.5 to 36.3 billion in the respective periods, among other horticultural products, apparel and clothing accessories, the value of imports still outweighs the import earnings. In 2017Q3, the imports value grew by about 20.3% from KSh 374.8 to KSh 450.9 billion in 2016Q3, attributed mainly to heavy imports of sugar, maize, petroleum products, and industrial machinery. For instance, industrial machinery imports had the highest proportion of expenditure (13.2% of the total import bill) in 2017Q3 despite recording a decline of about 8.8% in the same period (KNBS statistical release, 2017). Given the overall negative trend, Kenya's ability to meet external financial needs continues to sink. Thus, Kenya can only lower the deficit in the BoP by shifting to an industrialized economy.

4.2 Stationarity Test

The use of non-stationary data in time series analysis has always been criticised since it leads to spurious results. In the current study, we employed ADF (Augmented Dickey-Fuller) test to evaluate the stationarity of the data. The ADF test tests the hypothesis (H_0) that data is not stationary versus an alternative hypothesis (H_1) which states the data is stationary. The results indicated that the exchange rate data is stationary at level, at a 5% level (ADF = -3.845, p = 0.044 < 0.05) (Table 2).

Table 2: ADF test for Exchange Rate Data

Variable	Series	ADF statistic	Lag order	p-value	Comment
Exchange rate	At level	-3.485	6	0.044	Stationary
	Log differenced	-7.555	6	0.010	Stationary
BoP	At level	-3.342	6	0.065	Not Stationary
	Log differenced	-9.175	6	0.010	Stationary

The ADF test results for the BoP data showed that the null hypothesis of non-stationarity should not be rejected (should be accepted) at a 5% significance level (ADF = -3.342, p = 0.065 > 0.05) (Table 13). In contrast, the first difference series was stationary at a 5% level (ADF = -8.532, p = 0.01 < 0.05), fitted the study's asymmetric heteroscedastic models led to convergence problems; hence no parameters were estimated when fractional differencing as specified in the ARFIMA (p, d, q) model in the RUGARCH package in R (R Core Team, 2016). While the exchange rate data is stationary, at level, it has an upward trend. Besides, there was a downward trend in the monthly BoPs series over the study period, as shown in Figure 1. To obtain data that is stationary, the monthly first difference of the logarithms of the absolute series for exchange rate and BoP data were computed using the formula in equation 4.1

$$D.\log(X_t) = \ln|X_t| - \ln|X_{t-1}| \quad (4.1)$$

The resultant series are visualized in Figure 2. The two series are more stationarity than their respective mother series depicted in Figure 1.

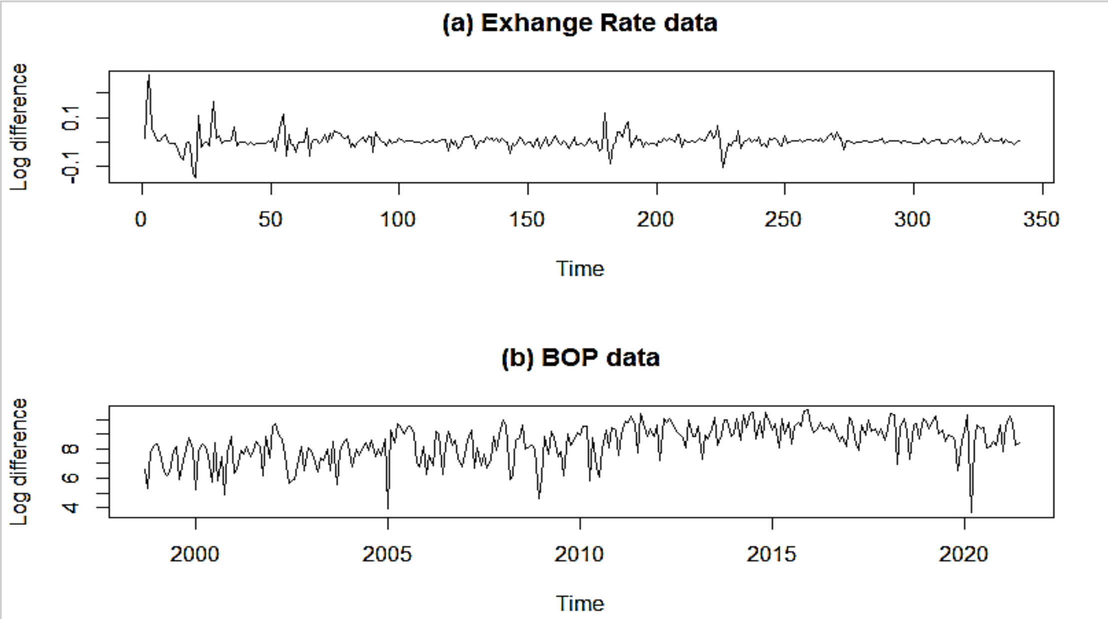

Figure 2: Time plot of log differenced series for Kenya’s exchange rate and BoP data

4.3 Descriptive Statistics

Table 3 displays the descriptive statistics and normality test of Kenya's exchange rate and current account BoP data and their respective log-differenced series over the study period. The skewness statistic for the exchange rate data is close to zero, indicating that the data approximately follows a normal distribution. The negative kurtosis (-0.599) suggests that the data is not heavy-tailed; instead, the outliers are less extreme than that of a normal distribution (Westfall, 2018). The low-value kurtosis statistic specifies that the series is slightly platykurtic. However, the log differenced series of the exchange rate data has more interesting properties associated with the stylised facts of the financial series. The series mean is close to zero and has excess kurtosis. As a result, the study proposed that the asymmetric GARCH model can be more appropriate.

The skewness statistic for Kenya's current account BoP is –0.236, signifying that the lower tail of the distribution is slightly thicker than the right tail. The negative excess kurtosis (-1.416) suggests that the data is not heavy-tailed and that hence the outliers are less extreme than that of a standard distribution (Westfall, 2018). Like the log-differenced series of Kenya's exchange rate, the log-differenced series of Kenya's BoP data is heavy-tailed (Kurtosis = 9.541) compared to its parent series (-1.416). Thus, the log-differenced series in each case will be studied.

Table 3: Descriptive statistics and normality test

	Descriptive Statistics							Shapiro-Wilk test (SW)	
	N	Min	Max	Mean	SD	Skewness	Kurtosis	Statistic	p-value
Exchange Rate	342	35.92	110.14	79.26	16.49	0.000	-0.599	0.967	0.000
D.log (EXCHR)	341	-0.151	0.274	0.0332	0.0332	2.432	24.841	0.7014	0.000
BoP	275	-11,9462.7	-2,175.2	-49,244.1	35,629.9	-0.236	-1.416	0.904	0.000
D.log (BoP)	274	-2.02	1.33	0.0102	0.329	-0.411	9.541	0.923	0.000

Note. EXCHR is the exchange rate

One of the important features in fitting GARCH type models is the normality assumption. As statistically quantified by Shapiro-Wilk's (SW) test indicates that all the series are not normally distributed (all $p < 0.05$). The histograms in Figure 3 indicates that the series slightly deviated from normal. Given that the data doesn't follow a standard distribution, mainly caused by the flatter tail than those of a normal distribution, it is necessary to use non-normal distributions, for instance, the student t-distribution (std), GED, and skewed student distribution ("sstd") or skewed normal distribution (snorm) (Baillie & Bollerslev, 1989; Bollerslev, 1987; Beine *et al*., 2002). Given that the data slightly deviates from normal, the current study adopted the snorm when fitting all the potential asymmetry GARCH models for the log-differenced series of Kenya's exchange rate and BoP data.

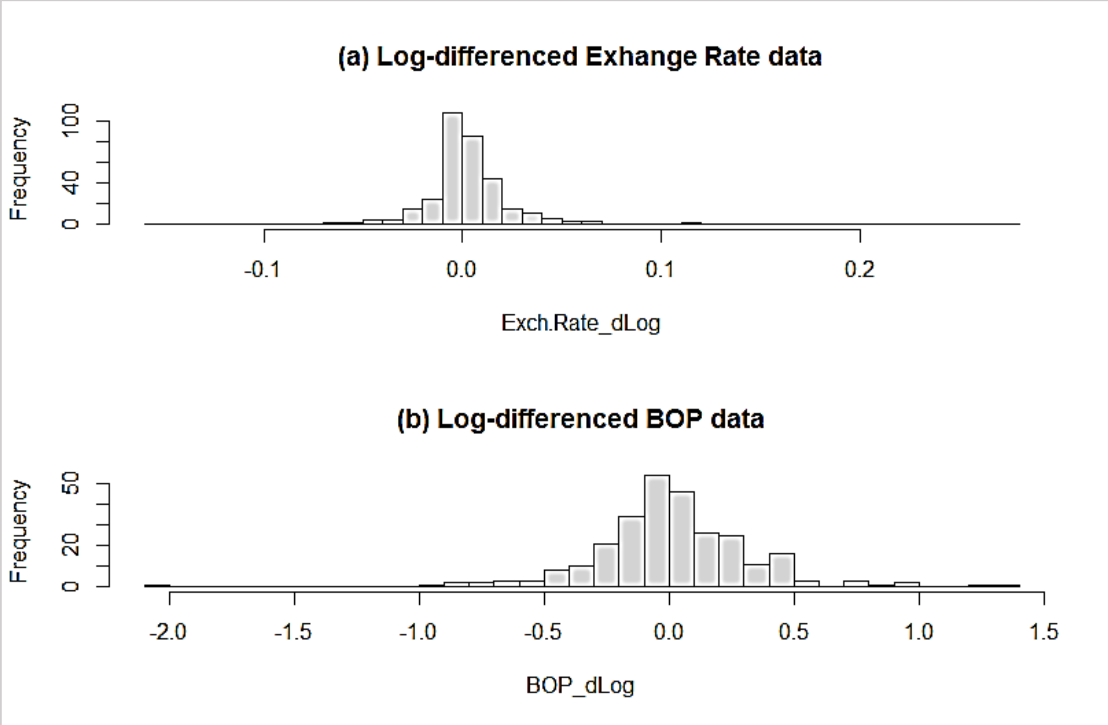

Figure 3: Histogram of log-differenced series of Kenya's exchange rate and BoP data.

4.4 Testing for ARCH Effects

The GARCH type model is an extension of the ARCH-type models. The GARCH (p, q) model points towards the ARCH (r = q + p) model. Thus, the preliminary stage to fitting the model entails testing for the existence of ARCH effects. The Lagrange Multiplier (LM) test was employed to examine the existence of ARCH effects on the squared residuals of the AR (p) model. The null hypothesis state that there is no presence of ARCH properties. Table 4 displays the test results.

Table 4: Arch effects test for the log-differenced series of exchange rate and the BoP data

Series	N	Ch-Square	Degree of freedom	p-value
D.log (EXCHR)	342	73.86	12	0.000
D.log (BoP)	275	100.5	12	0.000

Regarding the log-differenced series of exchange rates, the resultant LM statistic at 12 degrees of freedom is 73.86 with an associated p-value less than 0.01. Therefore, the null hypothesis (H0) should not be accepted (rejected) at a 1% level of significance, indicating strong evidence of the existence of ARCH effects. The LM was also applied to test for the existence of the ARCH effect on the squared errors of an AR (p) process.

The resultant LM test results indicate that the H0 is to be rejected at a 1% significance level (Chi-squared (12) = 100.5, p < 0.01), supporting the presence of ARCH effects in the series. The findings justify the use of the GARCH type models in both series. On the same note, the GARCH (Bollerslev, 1986) is an extension of the ARCH family, where σ_t^2 It depends on the lags and lags of the squared error term. The GARCH model is an Autoregressive Distributed Lag (ADL) (p, q) model hence likely to provide more parsimonious parameterisations than the ARCH model.

4.5 Model Selection and Specification

4.5.1 Mean equation Selection

The mean equation in asymmetric GARCH models is an ARMA process. The mean equation selection for the log differenced series of exchange rates and BoP data are discussed concurrently.

4.5.1.1 Exchange Rates

One of the visualisation tools that help examine the presence of autocorrelation in time series data is ACF; as shown in Figure 5, there is a significant autocorrelation coefficient with some seasonality, which repeats itself after some interval k.

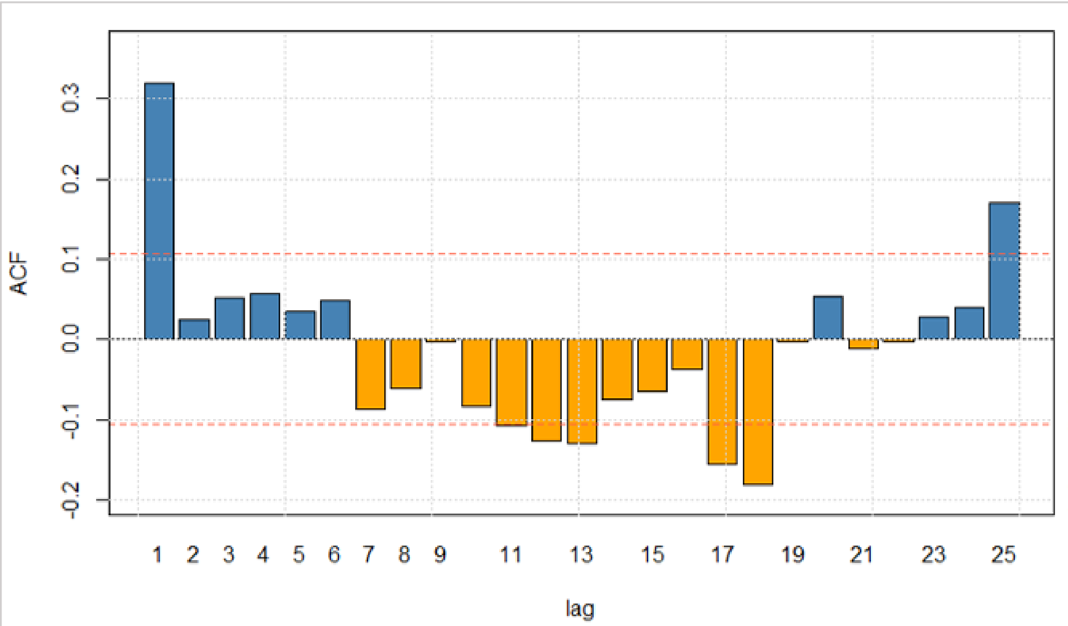

Figure 4: ACF of log differenced series of Kenya’s exchange rate data

Figure 4 shows the log differenced series of exchange rate data alongside its decomposed time-series properties and ascertains that the series has seasonality.

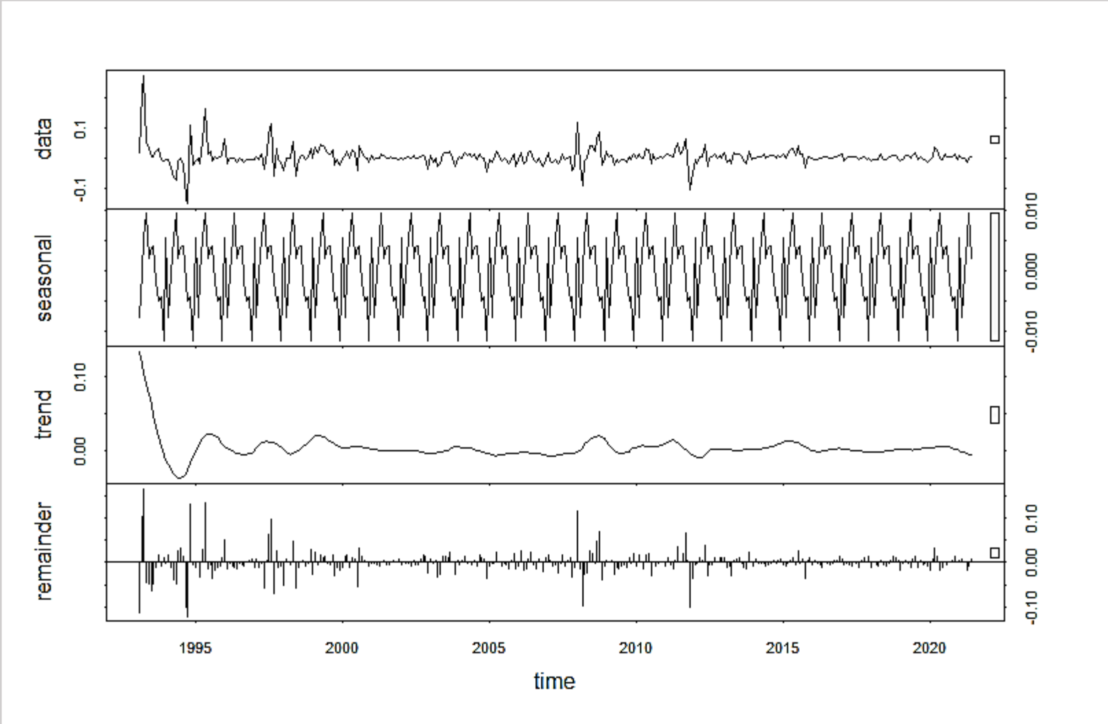

Figure 5: Time series decomposition of log-differenced series of exchange rates

One of the notable challenges with modelling with the *ugarchspec* function in the Rugarch package in R (R Core Team, 2016) is that it does not account for the seasonality aspect in the ARMA mean equation model. Based on the minimisation of AIC, and BIC values the $auto.arima$ function in the $fpp2$ package in R suggests that ARIMA (4,0,0) (2,0,0) [12] with non-zero mean could be the best Seasonal ARIMA (SARIMA) model (AIC = -1385.34, BIC = -1354.68). The model shows that the mean equation model should capture seasonality. Yet, the RUGARCH package does not account for seasonality in the asymmetric modelling producer. The seasonal argument was set to False to account for seasonality assuming the series has no seasonal component to incorporate the Fourier terms in the model, as in Andersen and Bollerslev (1997). The resultant models incorporate the ARMA and Fourier terms as external regressors. K was specified as from 1 to 6 since K should not be greater than period/2. Given that the periodicity of the series is 12; $1 \leq k < 6$. All the ks favoured ARIMA (3,0,0). Therefore, the mean equation in the *ugarchspec* function in the Rugarch package will be ARMA (3,0). Besides, the best fit GARCH had the parameters p and q as one. The optimal ARMA (3,0) is akin to AR (3) process, indicating autocorrelation in the series.

4.5.1.2 Balance of Payment (BoP)

Similarly, the ACF of the log difference series of BoP shows some significant autocorrelation coefficients that tend to repeat after some interval. The behaviour of the ACF suggests that a SARIMA process best fit the series.

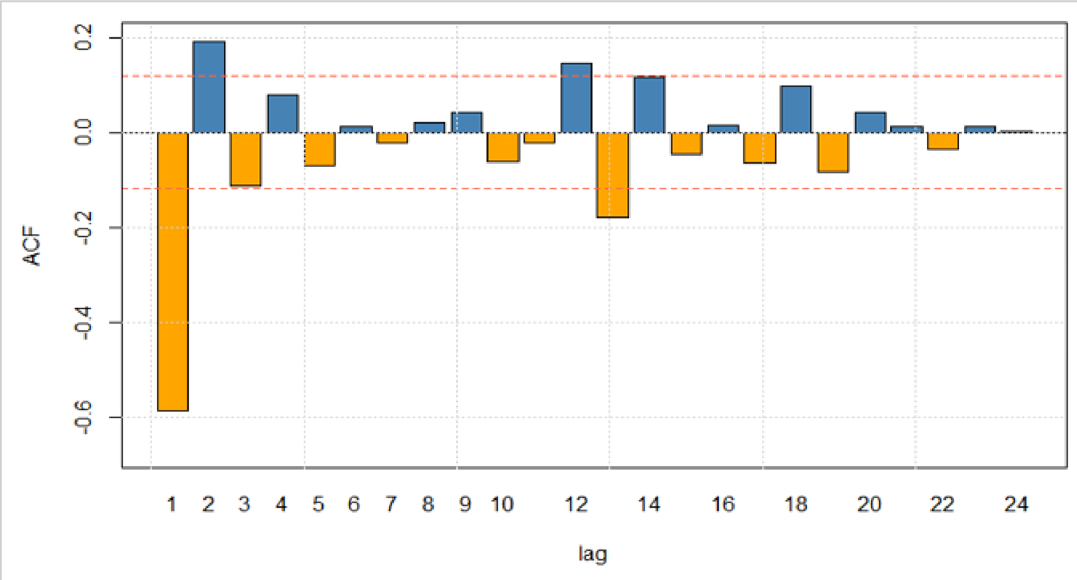

Figure 6: ACF plot of log-differenced series of BoP data

Figure 6 shows the log differenced series of BoP rate data alongside has seasonality. The best SARIMA model ascertains the feature to the data specified as ARIMA (4,0,0) (2,0,0) [12] with a non-zero mean (AIC = 33.7, BIC =62.61). Thus, it is vital to account for such seasonality using Fourier terms to identify an ARMA mean equation.

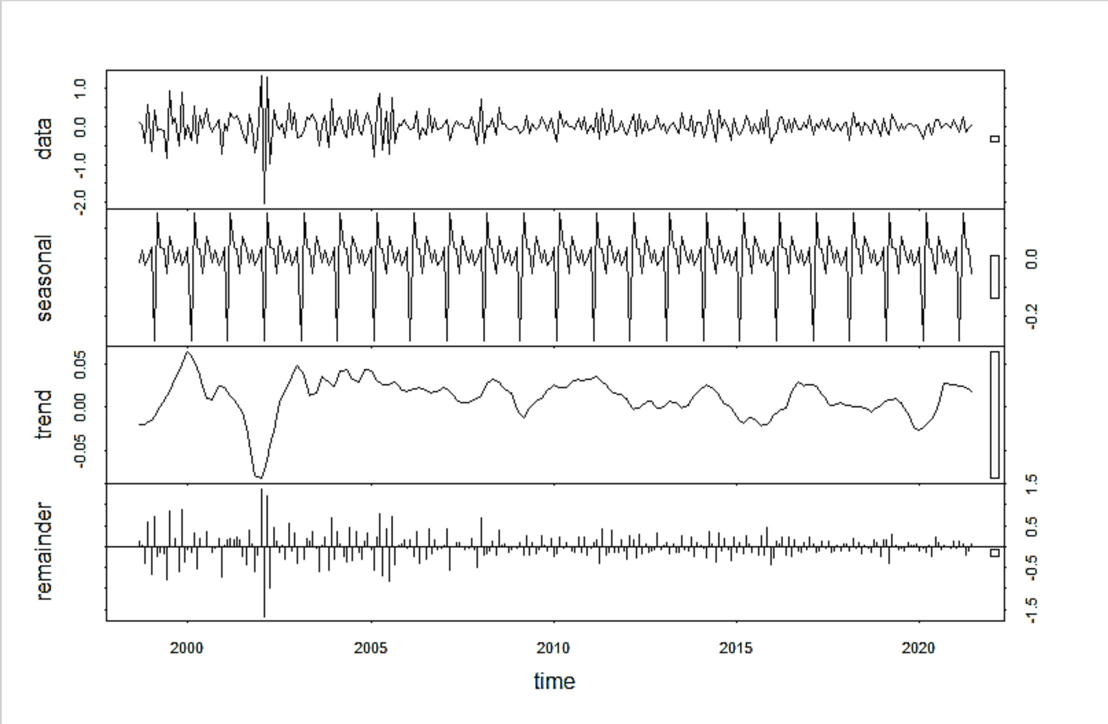

Figure 7: Time series decomposition of log-differenced series of BoP data

Table 5 shows the mean equation model selection for the log-differenced series of BoP data. The two ARIMA (p, q) models have a balanced preferential. For instance, ARMA (0,3) is favoured by the AIC (20.65) with a k of 4. Yet, ARMA (1,1) is preferred by the BIC (46.11), and AIC corrected (AICc) (21.87) with a k of 1 and 3, respectively. Thus, both mean equations were used, and the best-fit equation was determined based on the resultant model's performance.

Table 5: Mean equation model selection by incorporation of Fourier terms

k	Model	Constant	AIC	AICc	BIC
1	ARIMA (1,0,1)	24.7456	24.43	24.75	46.11
2	ARIMA (0,0,3)	26.4645	25.78	26.46	58.3
3	ARIMA (1,0,1)	21.8689	21.03	21.87	57.16
4	ARIMA (0,0,3)	22.0508	20.65	22.05	67.62
5	ARIMA (0,0,3)	23.1414	21.28	23.14	75.48
6	ARIMA (1,0,1)	23.5359	21.68	23.54	75.87

Note. All the Mean equation models had non-zero mean indicated by the constant; AICc = AIC corrected

4.5.2 GARCH Model Selection

4.5.2.1 Exchange Rates

Table 6 presents possible ARIMA (3, 0) – GARCH (1, 1) models for the log-differenced series of exchange rate data alongside their evaluation metrics (AIC and BIC). The optimal variance equation based on the minimization of the two metrics was APARCH (1,1) - ARMA (3,0) model with a skewed normal distribution (AIC = -4.6871, BIC = -4.5860).

Table 6: Model Selection for the log-differenced series of exchange rate data

GARCH Model	Mean Model	AIC	BIC	LL	ME	RMAE
EGARCH (1,1)	ARMA (3,0)	-4.4492	-4.3593	766.5814	-0.0006	0.1304
IGARCH (1,1)	ARMA (3,0)	-4.6394	-4.5720	797.0147	0.0034	0.1334
APARCH (1,1)	ARMA (3,0)	-4.6871	-4.5860	808.1503	0.0020	0.1340
TGARCH (1,1)	ARMA (3,0)	-4.1037	-4.0138	707.6726	-0.0055	0.1422
GJR-GARCH (1,1)	ARMA (3,0)	-4.6285	-4.5386	797.1539	0.0029	0.1290

The findings differ from those of Petrică and Stancu (2017), who established that AR (3) - EGARCH (2, 1) was the best model fit to the best model for estimating daily returns of EUR/RON exchange rate. While the mean equation is similar, the GARCH model is different. The findings can be associated with methodological disparity. The current study specified a skewed normal distribution, unlike Petrică and Stancu (2017), who preferred students' t-distribution (AIC $\approx$ 0.7880). Apart from using a different reference currency, the frequency of the series was high (daily EUR/RON exchange rates) and spanned 4th January 1999 to 13th June 2016. Unlike their study, the current study used monthly KES/USD exchange rates which portrayed different time-series properties. In the current study, ARMA (3,0) - APARCH (1,1) is the best fit model since APARCH of Ding *et al*. (1993) accounts for leverage and the Taylor effect (Taylor, 1986), which postulates that the observed that the sample autocorrelation of absolute returns was larger than from squared returns.

Table 7 summarizes the optimal parameters for the best model ARMA (3,0). - APARCH (1,1). While determining the optimal order, ω was fixed to 0.000020 and was determined based on automatic optimization of the parameter based on the *garchFit* () function in the Rugarch package (R Core Team, 2020). From the results, ϕ_3 of the AR (3) mean equation process is -0.151835 and is statistically significant (p = 0.000), indicating a negative autocorrelation. Aggregately, the three AR parameters suggest that the exchange rate is characterized mainly by trends, as indicated in the decomposition of the series in Figure 5. As shown in Figure 4, the log differenced series of exchange rate data has a decreasing trend component.

Table 7: Optimal Parameters

Parameter	Estimate	Std. Error	t value	p-value
mu	0.001532	0.00089	1.710710	0.087135
AR1	0.072017	0.076051	0.946951	0.343664
AR2	-0.043664	0.065509	-0.666541	0.505065
AR3	-0.151835	0.042632	-3.561516	0.000369
Omega (fixed)	0.000020			
alpha1	0.649600	0.133069	4.881666	0.000001
beta1	0.206667	0.077380	2.670788	0.007567
eta11	-0.000005	0.071421	-0.000065	0.999948
Lambda	2.475383	0.051397	48.162375	0.000000
skew	1.128859	0.051947	21.731141	0.000000

Note. restricted Variance Dynamics: GARCH Model: APARCH (1,1); Mean Model: ARMA (3,0); Distribution: snorm

Algebraically, the square residuals models can be represented as follows:

$$\sigma_t^2 = 0.000020 + 0.0720X_{t-1} - 0.0437X_{t-2} - 0.15184X_{t-3} + 0.6496\,|\epsilon_{t-1}| + 0.2067\,\sigma_{t-1}^2 - 0.000005v_{j1} + 2.47538\,\epsilon_{t-1}$$

The GARCH parameters were approximated to be α_1= 0.6496, β_1=0.2067, $\eta_{11} = -0.000005$, $\lambda = 2.4754$.

A key stylized fact of financial time series data that GARCH models capture is volatility clustering. The persistence parameter captures the feature $\hat{P}$. For the APARCH model, the persistence model is estimated using equation 4.3

$$\bar{P} = \sum_{j=1}^{P} \beta_j + \sum_{k=1}^{q} \alpha_j k_j \quad (4.3)$$

Given that α_1 and β_1 is statistically significant at a 1% level ($p < 0.01$), there is a persistent volatility clustering in the series. Regarding volatility persistence, the research findings revealed a small value of the persistence parameter (β) hence a rapid decrease of the increase in the conditional variance due to shocks.

Besides, the significance of $\lambda = 2.4754 > 0$ at a 1% significance level (SE = 0.051397; $p < 0.01$) suggest presence of a statistically significant leverage effect. The leverage effect is a common feature in financial time series, where large negative past observations of α_t increases volatility more than positive past observations of a similar magnitude. The non-zero leverage parameters ascertain the presence of asymmetry in the exchange rate series. Conventionally, negative leverage parameters indicate an asymmetric reaction for positive returns in the conditional variance equation. In contrast, positive leverage parameters indicate that negative shocks or bad news increase volatility (Petrică & Stancu, 2017).

Thus, the resultant positive coefficient of λ (positive asymmetry) shows the absence of leverage result in the exchange rate series. Instead, volatility is positively correlated with the series. In the current finding, positive shocks on the exchange rate generate higher volatility than negative shocks of equal magnitude; other factors are kept constant. Despite inconsistent results with theory, there is always possible that empirical evidence deviates from the theoretical perspective. For instance, while modelling volatility or return series Nigerian, Onwukwe, Bassey & Isaac (2011) found a positive coefficient of λ in the UBA, Mobil, and Unilever returns with returns for the Guinness stock prices only indicating the presence of leverage effect. In another study, BENEDICT (2013) demonstrated the absence of the leverage effect in Ghana's monthly inflation. The study of Nwoye (2017) also revealed the presence of volatility clustering, leverage effect, and asymmetric effect in the Nigeria's Exchange rate against the USD spanning January 1999 to December 2012, though using the EGARCH (2,2) as the best fit model. Petrică & Stancu (2017) also found the presence of positive and negative asymmetry in the returns of EUR/RON exchange rate while using the AR (3) - EGARCH (2, 1) model.

Besides, Pahlavani & Roshan (2015) found a positive leverage effect in the exchange rate of Iran (IRR/USD) using that ARIMA (7,2), (12) – EGARCH (2,1) was the best fit model. Contrarily, Thorlie, Song, Wang, & Amin (2014) found negative asymmetry in the Sierra Leone/USA dollars exchange rate returns computed from the monthly data from January 2004 to December 2013 while using asymmetric GJR-GARCH models. In Kenya, Omari, Mwita, & Waititu (2017) found asymmetry and presence of negative leverage effect in Kenya's daily exchange rates spanning 3rd January 2003 to 31st

December 2015 while using the AR (2)-E-GARCH (1, 1) and AR (2)-GJR-GARCH (1, 1). The findings vary due to the different time frames used to estimate volatility.

4.5.2.2 Balance of Payments

Having determined that two mean equations can best fit model the log differenced series for BoP data, the next stage best fit GARCH had the parameters p and q Table 8 presents a combination of several ARIMA (p, q) – GARCH (p, q) models alongside their evaluation metrics (AIC and BIC). The optimal variance equation based on the minimization of the two metrics was ARMA (1,1) - IGARCH (1,1) model estimated assuming a skewed normal distribution (AIC = -0.14475, BIC = -0.07882).

Table 8: Model Selection for the log-differenced series of BoP data

GARCH Model	Mean Model	AIC	BIC	HQ	LL	ME	RMAE
EGARCH (1,1)	ARMA (0,3)	-0.126487	-0.020994	-0.084145	25.3287	0.002181	0.425351
EGARCH (1,1)	ARMA (1,1)	-0.12188	-0.02958	-0.08483	23.69797	0.002107	0.426163
IGARCH (1,1)	ARMA (0,3)	-0.12911	-0.04998	**-0.09735**	23.68753	0.002115	0.425377
IGARCH (1,1)	ARMA (1,1)	**-0.14475**	**-0.07882**	-0.11829	24.83077	0.001885	0.426304
APARCH (1,1)	ARMA (0,3)	-0.12934	-0.01066	-0.08171	**26.71969**	0.002008	0.425542
APARCH (1,1)	ARMA (1,1)	-0.12692	-0.02143	-0.08458	25.3878	0.001856	0.426319
TGARCH (1,1)	ARMA (0,3)	-0.13203	-0.02653	-0.08968	26.08743	0.002134	**0.425515**
TGARCH (1,1)	ARMA (1,1)	-0.12742	-0.03512	-0.09037	24.45693	0.001991	0.426302
GJR-GARCH (1,1)	ARMA (0,3)	-0.13602	-0.03053	-0.09368	26.63447	0.002001	0.425528
GJR-GARCH (1,1)	ARMA (1,1)	-0.13420	-0.04190	-0.09715	25.38583	**0.001854**	0.426322

Table 9 summarizes the optimal parameters for the ARMA (1,1)- IGARCH model. Like when estimating the best fit model for the exchange rate data, the optimal parameter, μ was fixed to 0.009617 determined based on automatic optimization of the parameter based on the *garchFit()* function in the Rugarch package (R Core Team, 2020).

Table 9: Optimal Parameters

Parameter	Estimate	Std. Error	t value	p-value
mu (fixed)	0.00962			
AR1	-0.1196	0.0870	-1.3746	0.1692
MA1	-0.6576	0.0702	-9.3671	0.0000
Omega	0.0002	0.0005	0.3779	0.7055
alpha1	0.0881	0.0515	1.7094	0.0874
beta1	0.9119	NA	NA	NA
skew	0.7379	0.0654	11.2865	0.0000

Note. Conditional Variance Dynamics: IGARCH Model: IGARCH (1,1); Mean Model: ARMA (1,1); Distribution: ger

Algebraically, the square residuals of ARMA (1,1) - IGARCH (1,1) can be represented in equation as follows.

$$\sigma_t^2 = 0.000195 - 0.11959\mathrm{X}_{t-1} - 0.657601\mathrm{X}_{t-1} + 0.0881\varepsilon_{t-1}^2 + 0.9119\sigma_{t-1}^2$$

The inclusion of AR (p) process parameters indicates the presence of negative autocorrelation in the series. The maximum specification of one lag in the Autoregressive part of the mean model shows that the BoP is characterized mainly by trends. As indicated in Figure 7, the BoP has a decreasing trend. The GARCH parameters were approximated to be α = 0.0881, β = 0.9119. The conditional variance parameters are not statistically significant at a 1% level (associated p-values > 0.05), indicating the absence of (persistent) volatility clustering in the series. Unlike in the log differenced exchange rate series in Figure 5, the log differenced series shows three clusters prior to 2007 and after that depicts a constant (mean) and variance over time. Thus, the IGARCH model is well suited due to its parametrization algorithms. The IGARCH model assumes that the persistence parameter $\bar{P}$ is one and is thus imposed during the estimation procedure such that unitary persistence other unconditional variance parameters are not computed (Engle & Bollerslev, 1986). As such, the absence of the leverage parameter indicates that BoP does not depict the leverage effect.

BoP can be seen as an aggregate measure of international flows in a country. Its growth or changes seem to be additive over time since it has so many parameters. Good or bad news in one sector, such as the goods market, might be alleviated by stability in another sector, such as the service sector. Contrarily, exchange rates seem unilateral, and one external shock such as wars or sanctions can cause an immediate shock in the exchange rate. Such shocks tend to recur after some interval, such as the electioneering period in Kenya, hence the clustering effect. Therefore, BoP can only display multiplicative or additive trend parameters over time with less clustering. Figure 1 (b) shows that BoP data has a multiplicative decreasing trend with no significant clusters.

4.6 Residual Diagnostics

The adequacy of any fitted model is examined to detect any misspecification error. The current study tested whether residuals from the best fit model for the exchange rate data meet the normality assumption, show no serial association, and have no ARCH properties. The key aim is to examine whether the model minimizes the prediction errors.

4.6.1 Exchange rates

Figure 8 shows the graph of the residuals from the ARMA (3,0)- APARCH (1,1) alongside the histogram. It shows clear evidence that the residuals $\{\epsilon_t\}$ mimics a Gaussian white noise.

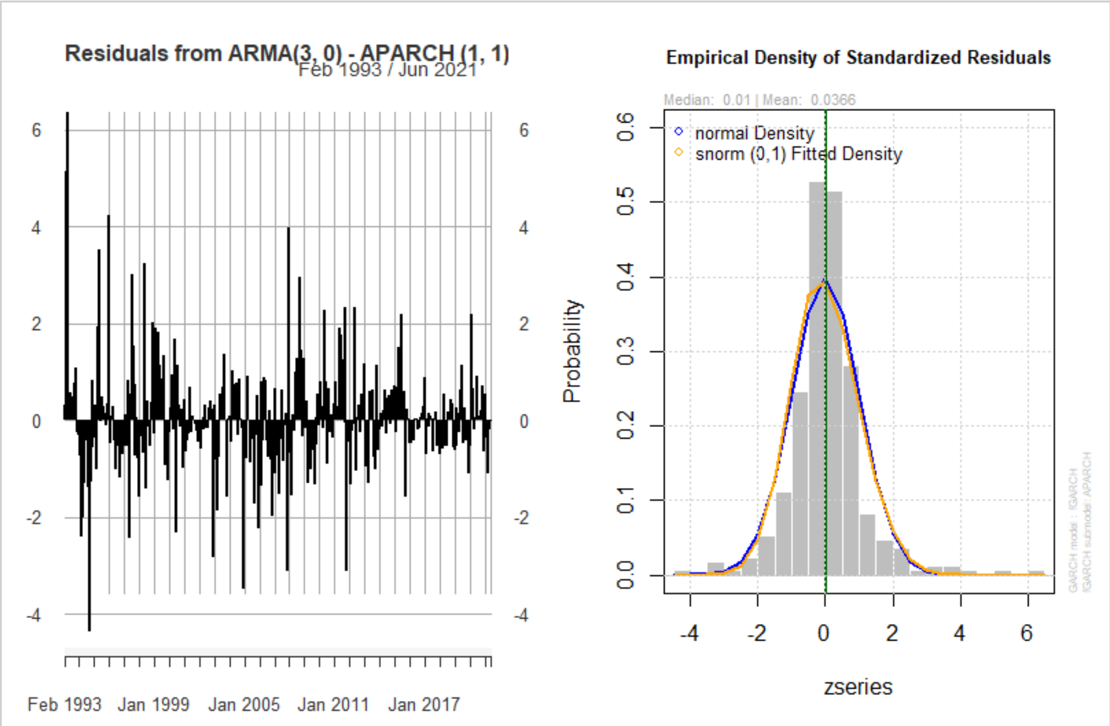

Figure 8: Plot Residuals (left) from ARMA (3,0) -APARCH (1,1) model alongside an Empirical Density of Standardized Residuals.

Besides, there are few significant autocorrelation coefficients in the resultant model residuals, as depicted in Figure 9.

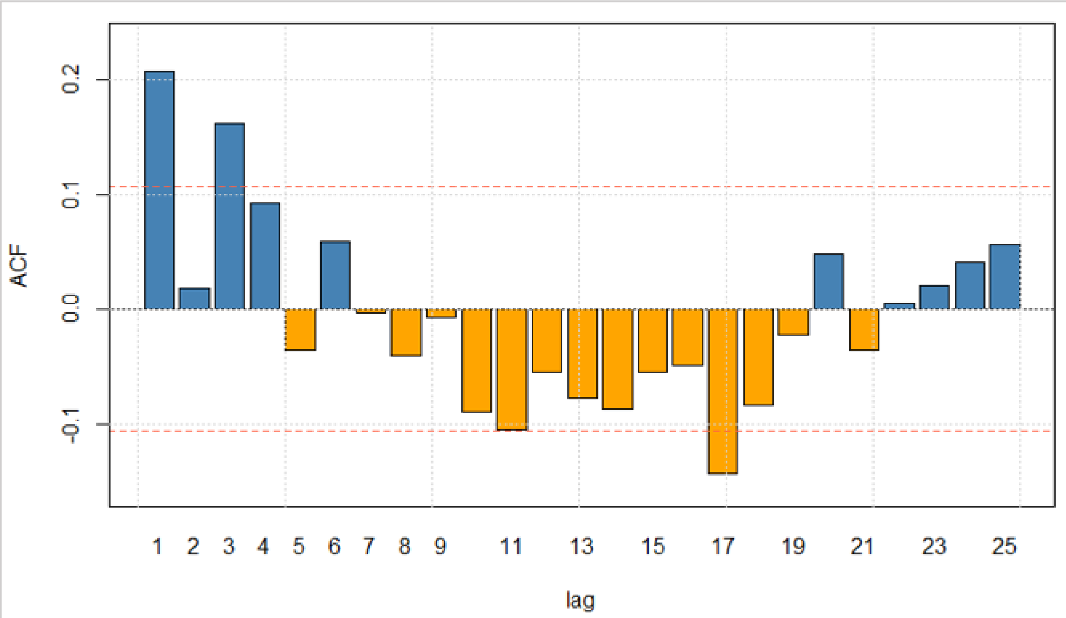

Figure 9: ACF plot of residuals from ARMA (3,0) -APARCH (1,1) model

In a different statistical language, the residuals from the fitted model are purely random; hence the model is adequate. However, randomness is not the only necessary condition that must be satisfied. The residuals of an adequate model should not be autocorrelated. The Ljung–Box test was applied to the model residuals and the squared errors of the best fit ARMA (3,0) -APARCH (1,1) model for Kenya's exchange rate series. It tests the H0 of no serial association. In both cases, the Ljung–Box tests results indicated significant results (all p – values < 0.05) for all lags, evidence of the presence of serial correlation (Table 10). However, as shown by the ACF plot in Figure 9, there are few significant autocorrelation coefficients in the resultant model residuals hence has no serious implication in the fitted GARCH model.

Table 10: Weighted Ljung-Box Test

	Standardized Residuals		Standardized Squared Residuals	
	statistic	p-value	statistic	p-value
Lag [1]	14.81	0.000	17.50	0.000
Lag[2*(p+q)+(p+q)-1][8]	24.30	0.000	17.87	0.000
Lag[4*(p+q)+(p+q)-1][14]	29.29	0.000	19.20	0.000

Note. degrees of freedom = 3

Furthermore, it is also an expectation that ARCH-LM test results are non-significant. The LM test examines the H0 that “there are no ARCH effects.” The results in Table 11 show the absence of ARCH effects in the model residuals (all p-values > 0.05). Thus, the finding indicates that the fitted ARCH process was adequate.

Table 11: Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag [3]	0.2487	0.500	2.000	0.6180
ARCH Lag [5]	0.5832	1.440	1.667	0.8588
ARCH Lag [7]	0.8014	2.315	1.543	0.9436

The SBT, the NSBT, and the PSBT are usually carried out to evaluate the volatility model misspecification. Overall, the SB test evaluates the existence of leverage effects in the standardized residuals (to address the possibility of model misspecification) by predicting the squared standardized residuals based on lagged negative and positive shocks (Engle and Ng, 1993). Thus, it tests whether positive and negative shocks affect volatility differently from the fitted conditional heteroscedastic model. The NSBT examines if large negative shocks correlate with the volatility contrary to the fitted volatility model projection. On the contrary, the PSBT examines how larger positive shocks are associated with large biases in forecasted volatility. Hypothetically, the residuals are said to be $i.i.d$ because the model parameters ({β_i , $\underline{\beta'}$) in equations 3.11 and 3.12 are insignificant; hence the model is correctly specified. The fitted model SB and NSB tests are non-significant. However, the PSB is significant and, therefore, the joint effect at a 1% significance level (all p-values < 0.01) (Table 12). Thus, the squared residuals' significant positive reaction to shocks. Yet, the APARCH model has been designed to alleviate such biases (Ghalanos, 2022).

Table 12: Sign Bias Test

	t-value	p-value	Significance
Sign Bias	0.6441	0.052	Not Significant
Negative Sign Bias	0.1416	0.089	Not Significant
Positive Sign Bias	5.0158	0.000	Significant
Joint Effect	26.6334	0.000	Significant

4.6.2 Balance of Payment

Figure 10 depicts the residuals from ARMA (1,1) – IGARCH (1,1) alongside the histogram. It is evident that the residuals $\{\epsilon_t\}$ mimics a Gaussian white noise.

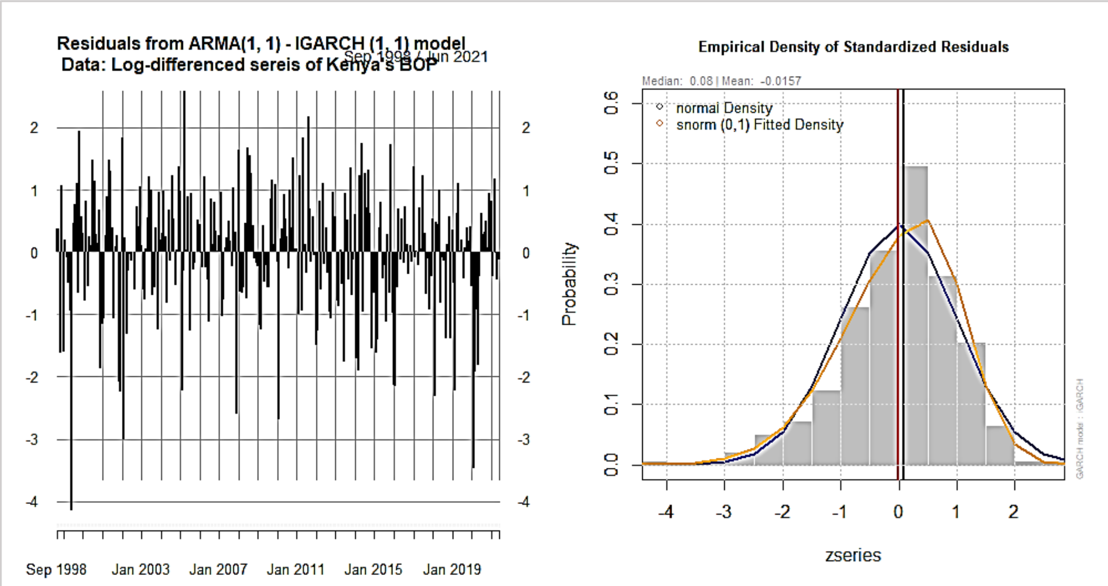

Figure 10: Plot Residuals (left) from ARMA (1,1) – IGARCH (1,1) model alongside an Empirical Density of Standardized Residuals.

Besides, there are few significant autocorrelation coefficients in the resultant model residuals, as depicted in Figure 9.

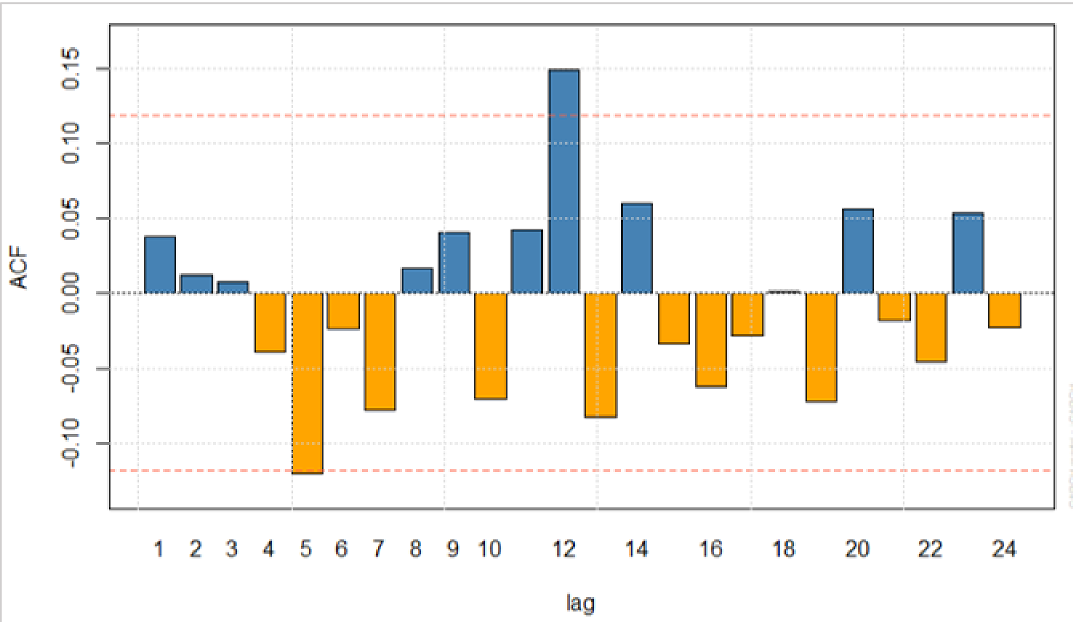

Figure 11: ACF plot of residuals from ARMA (1,1) -IGARCH (1,1) model

The Ljung–Box test was applied to the standardized squared residuals to test for serial association. In both cases, the test results were insignificant (all p – values > 0.05) for all lags included. The finding implies that there is no serial association; hence the model fits data well (Table 13).

Table 13: Weighted Ljung-Box Test

	Standardized Squared Residuals	
	statistic	p-value
Lag [1]	1.098	0.2947
$Lag[2 * (p + q) + (p + q) - 1][5]$	2.613	0.4823
$Lag[4 * (p + q) + (p + q) - 1][9]$	4.010	0.5871
Note. degrees of freedom = 2		

The third diagnostic test on the model residuals was examining for ARCH properties. The LM test examination showed that the H_0 that "there are no ARCH effects" should not be rejected (all p-values > 0.05) at each lag order. Thus, the results provide sufficient evidence of the absence of ARCH properties in the errors (Table 14).

Table 14: Weighted ARCH LM Tests

	Statistic	Shape	Scale	P-Value
ARCH Lag [3]	0.1337	0.500	2.000	0.7147
ARCH Lag [5]	1.9515	1.440	1.667	0.4821
ARCH Lag [7]	2.6837	2.315	1.543	0.5752

The SB test on the ARMA (3,0) - IGARCH (1,1) model fitted to exchange rate supports the nullity of negative and positive signs. However, there is a significant sign bias and joint effect at a 5% level (all p-values > 0.05) (Table 15). The evidence of sign bias indicates that the model can suffer from misspecification. However, the model is adequate as supported by the ARCH-LM test; hence can describe the volatility trends of the series. Besides, the model identified the absence of volatility clustering and leverage effect in the BoP series. In any case, there is no positive or negative bias.

Table 15: Sign Bias Test

	t-value	p-value	Significance
Sign Bias	2.4988	0.0131	Significant
Negative Sign Bias	0.0228	0.9818	Non-significant
Positive Sign Bias	1.5443	0.1237	Non-significant
Joint Effect	8.2911	0.0404	Significant

4.7 Estimating Volatilities

4.7.1 Exchange Rates

While the ARMA (3,0)-APARCH (1,1) model fitted to the log differenced exchange rate data fails the positive sign bias test, the model is adequately supported by the ARCH-LM test; hence can describe the volatility trends of the series. The in-sample volatilities were estimated as graphically presented in Figure 12. Overall, Kenya's exchange rate volatility appears to be converging over time, indicating sustained exchange rate stability. The GARCH α and β parameters fitted ARMA (3,0) - APARCH (1,1) model were statistically significant at a 1% level ($p < 0.01$), indicating a persistent volatility clustering in the series with rapid decelerating growth over time.

The first period depicts some notable occasions where volatility was high. The periods between 1993/94, 1995-1997, 2001/02, 2005/06, 2008/09, 2011-2013, and 2015/17 experienced high volatility and are associated with the electioneering period, which causes an incentive to both foreign and domestic investors due to uncertainty of the election outcome and likelihood of associated violence and disruption of business activities as witnessed in the aftermath of the 2007 elections. Besides, the higher volatility during 2008/09 can also be associated with the global financial crisis. The recent COVID-19 has also seen a sudden spike in the exchange rate volatility.

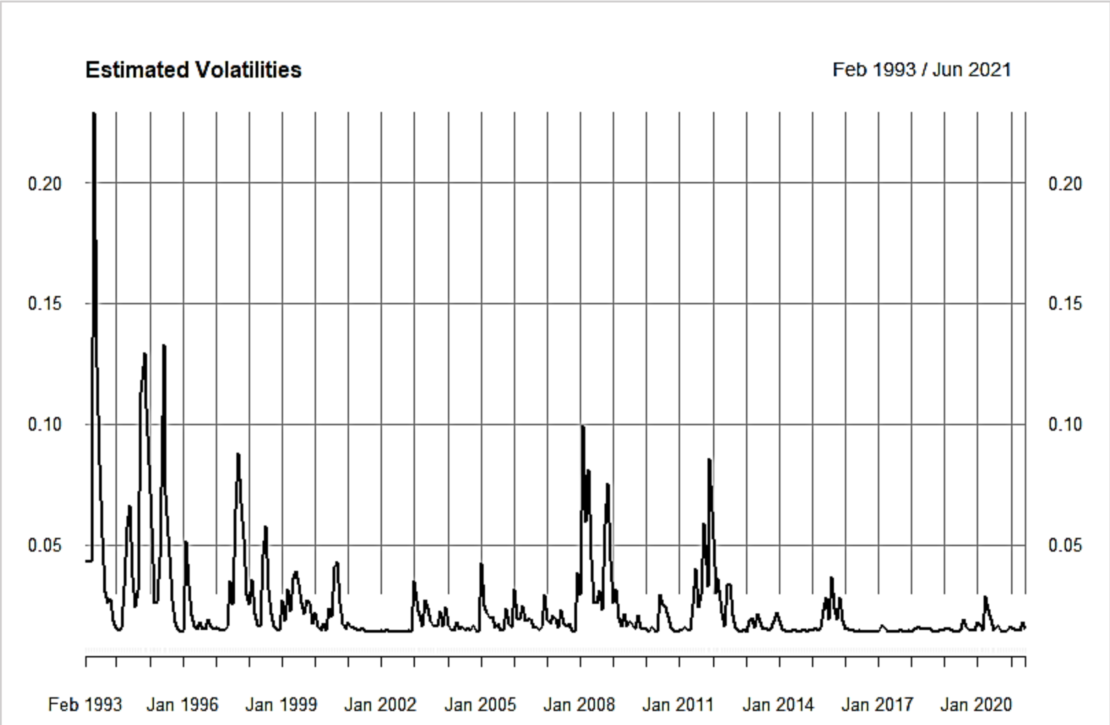

Figure 12: Estimate Exchange Rate Volatilities by ARIMA (3,0)-APARCH (1,1)

4.7.2 Balance of Payments

Given that the model residuals satisfy the post-diagnostic tests; the in-sample volatilities were estimated as graphically presented in Figure 13. Overall, Kenya's BoP volatility appears to be converging over time, indicating that the BoP has relatively stabilised over the past few decades. The volatility was relatively stable but still high before 2012. However, in 2011 volatility sharply rose and has since then remained high. The years 2014 and 2016 also recorded peaks in BoP volatility. Overall, from 2012 to 2019, had huge investments in the infrastructural investments in roads and railway transport. In addition, the BoP volatility significantly dropped towards the end of 2019 to mid of 2020.

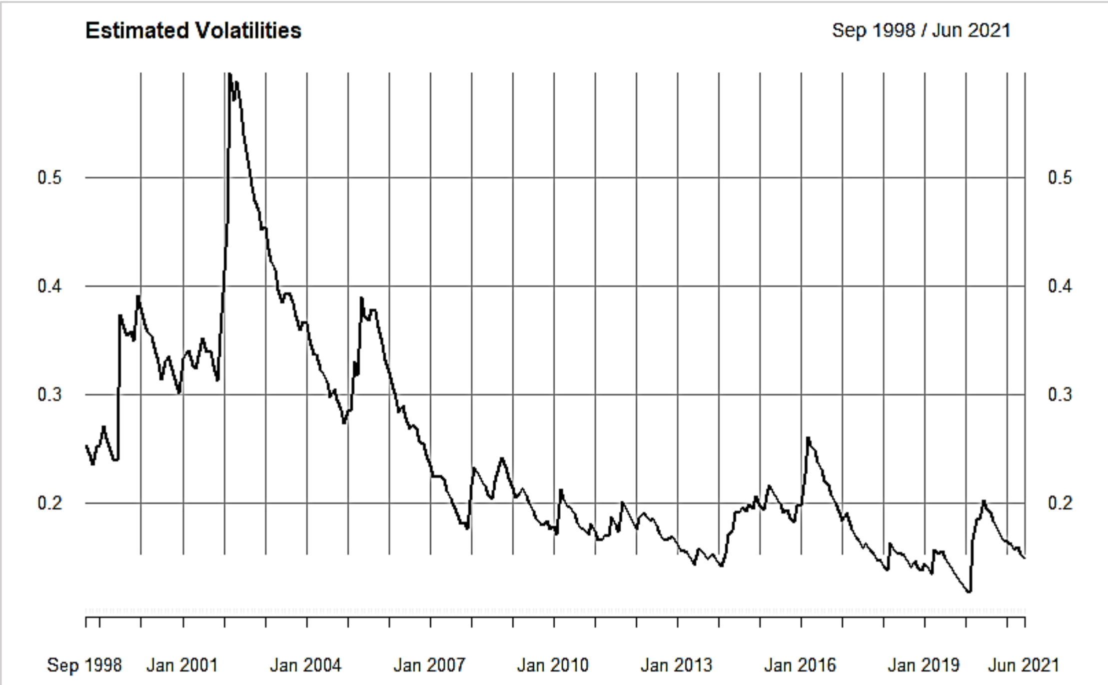

Figure 13: Estimated Kenya's BoP Volatilities by ARIMA (1,1)-IGARCH (1,1)

The sharp fall can be attributed to the global COVID-19 pandemic. The pandemic saw a restriction on international travel and merchandise trade. Besides, the high volatility can be attributed to Kenya's export and import basked composition. On one end, Kenya's import basket comprises highly valued finished or intermediate products such as electronics, industrial machinery, and road motor vehicles that keep appreciating over time. A change in lifestyle among Kenya's citizens towards materialism has seen a heavy influx of expensive automobiles and electronic devices. Yet, on the other end, the export basket comprises low-valued agricultural products, including tea, horticulture, apparel and clothing accessories, coffee, and Tobacco which are seasonal.

4.8 Forecasting

The ultimate in time series modelling is to make forecasts. The next sections present in and out-sample estimates of the exchange rate and BoP series log differenced series using their respective best fit model.

4.8.1 Exchange Rates

The ARMA (3,0) -APARCH (1,1) model satisfied the model adequacy test making it an appropriate forecasting model. First, an in-sample examination was evaluated where the actual values were superimposed with the estimated one standard deviation from the ARIMA (3,0)-APARCH (1,1) model (Figure 14). The confidence bands portray a similar series' pattern over time, providing reliable future forecasts.

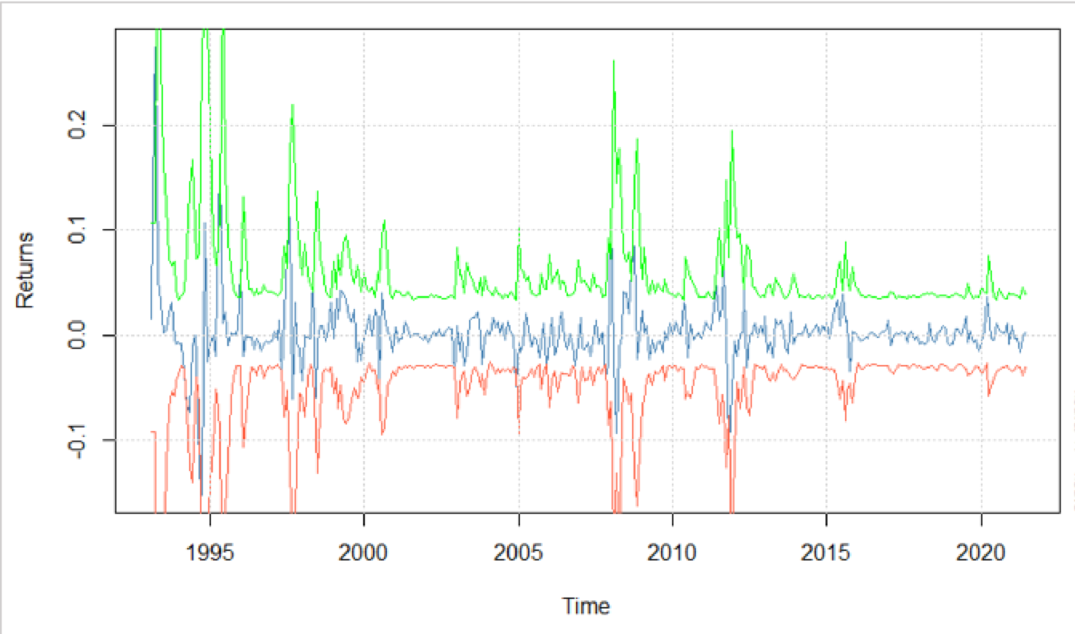

Figure 14: Actual Exchange rate values superimposed with one standard deviation confidence band estimated from ARIMA (3,0)-APARCH (1,1)

Figure 15 plots Kenya's log differenced exchange rates data from Jan 1993 to June 2021 with a 12-month step ahead of July 2021 to June 2022. The 99% confidence limits for the forecasts to account for volatility are included.

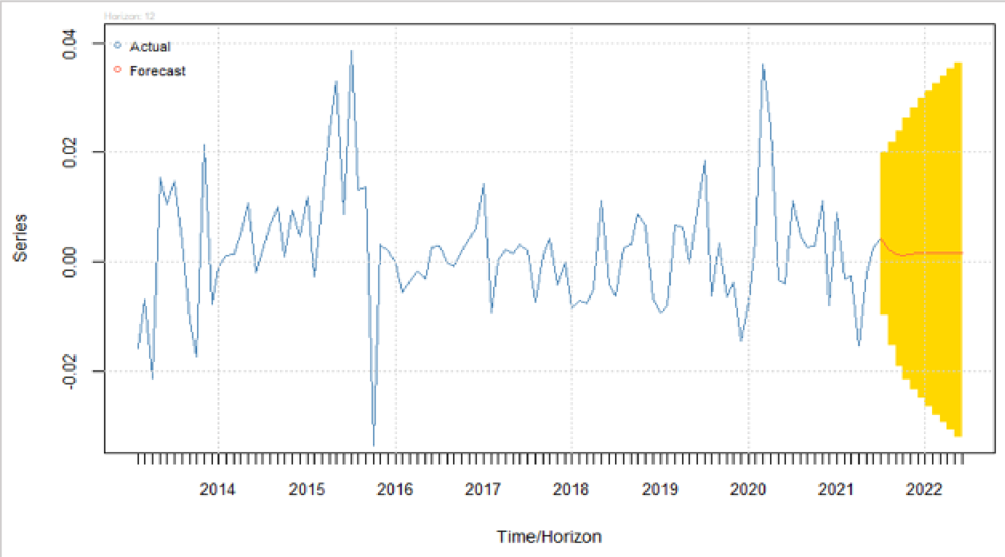

Figure 15: Log differenced exchange rates (Jan 1992 – June 2021) with a 12-month step ahead forecast with unconditional 1-sigma confidence bands.

The model predicts Kenya's exchange rate to remain relatively constant from July to June 2022. However, the volatility is increasing over time, as shown by the funnelled shape confident yellow band forecast limits widening out (Figure 15) due to increasing or diverging conditional standard deviation (σ) over time (Table 16). Thus, while the exchange rate can be somewhat stable and the diverging prediction intervals indicate that it is susceptible to future external shocks, which can either continue to weaken or strengthen it.

Table 16: GARCH Model Forecast

Forecast	Period	Forecasted Series	Sigma (Volatility)
T+1	July 2021	0.004329	0.01411
T+2	August 2021	0.002280	0.01773
T+3	September 2021	0.001330	0.02050
T+4	October 2021	0.001060	0.02280
T+5	November 2021	0.001393	0.02479
T+6	December 2021	0.001573	0.02658
T+7	January 2022	0.001613	0.02820
T+8	February 2022	0.001557	0.02969
T+9	March 2022	0.001524	0.03108
T+10	April 2022	0.001518	0.03238
T+11	May 2022	0.001527	0.03360
T+12	June 2022	0.001533	0.03477

Note. Forecast: T0=Jun 2021

4.8.2 Balance of Payments Data

The ARMA (1,1) - IGACRH (1,1) satisfied the diagnostic criteria of model parameter optimization and distribution of the model residuals. Forecasts can either consist of in-sample or out-of-sample forecasts. The in-sample estimates are commonly used to approximate the forecasted residuals hence model parameter selection. That is, parameters that minimise the in-sample prediction errors proxied by metrics such as AIC or BIC are optimal. A good way to visualize how a model minimizes the prediction errors is to superimpose the actual values with the estimated one standard deviation confidence limits from the fitted model, as illustrated in Figure 16. The confidence bands portray a similar pattern of the series over time with narrower gaps, given the scale of the data. Thus, the model is assumed to provide reliable forecasts.

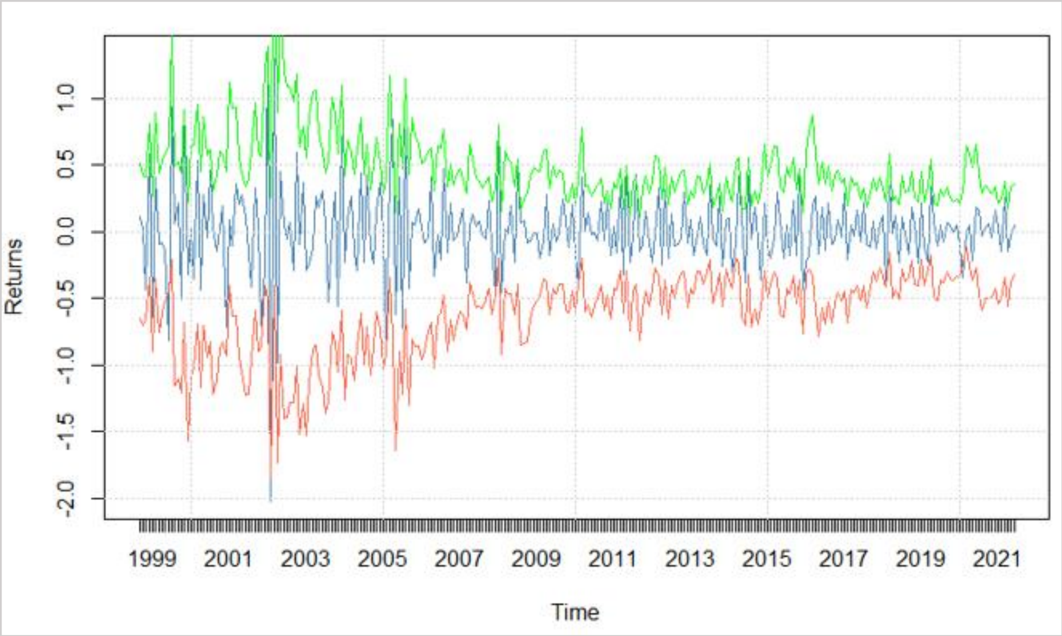

Figure 16: Actual BoP superimposed with one standard deviation confidence band estimated from ARMA (1,1)-IGARCH (1,1)

The out-sample forecasts can be done on both observed (resulting from splitting the data into a set) or unobserved (future estimates). The current study estimated the parameters using the entire sampled data points. Figure 17 plots Kenya's BoPs data from August 1998 to June 2021 with a 12-month step ahead of July 2021 to June 2022. The 99% confidence limits for the forecasts to account for volatility are included.

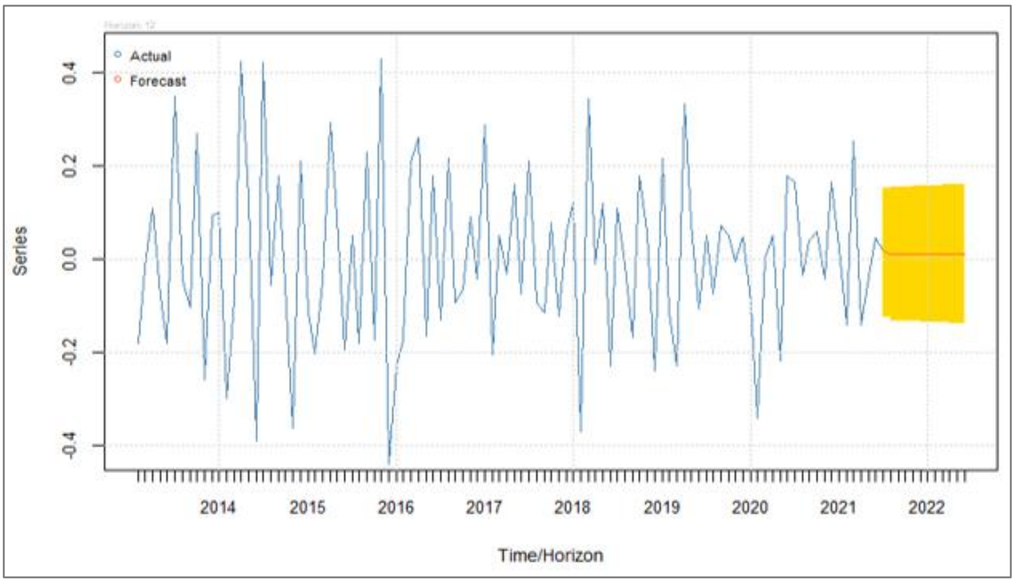

Figure 17: Actual balance of payment (Aug 1998 – June 2021) with a 12-month step ahead forecast with unconditional 1-sigma confidence bands.

The ARMA (1,1) - IGACRH (1,1) model predicted that Kenya's deficit in BoP is likely to remain relatively constant from December 2021 to June 2022. Besides, as measured by the restricted standard deviation (σ), the volatility is slightly decreasing but relatively constant over the same out-sample forecast period (Table 17), as shown by the rectangular shape confident yellow band forecast limits in Figure 17. The predictions are attributed to the findings that BoP has less volatility clustering than exchange rates which has significant volatility clustering. The trend can be attributed to the industrialization agenda of the government that was launched in 2017 and subsequent infrastructural investments in roads and railway transport.

Table 17: GARCH Model Forecast

Forecast	Period	Forecasted Series (Log differenced series)	Sigma (Volatility)
T+1	July 2021	0.016958	0.1426
T+2	August 2021	0.008739	0.1433
T+3	September 2021	0.009722	0.1439
T+4	October 2021	0.009605	0.1446
T+5	November 2021	0.009619	0.1453
T+6	December 2021	0.009617	0.1460
T+7	January 2022	0.009617	0.1466
T+8	February 2022	0.009617	0.1473
T+9	March 2022	0.009617	0.1479
T+10	April 2022	0.009617	0.1486
T+11	May 2022	0.009617	0.1493
T+12	June 2022	0.009617	0.1499

Note. Forecast: T0=Jun 2021

CHAPTER FIVE

SUMMARY, CONCLUSION, AND RECOMMENDATION

5.1 Summary

Volatility models are vital in the field of economics and funding. Estimating and forecasting volatility help make investment decisions, stock valuation, and financial or monetary policymaking. Linear time series models are not effective for explaining the features of a volatility series as they assume the existence of linear dependence in given series. Besides, most time-series data usually display volatility clustering resulting in the violation of the homoscedastic assumption of the equality of variance over time. Therefore, volatility models such as the symmetric and asymmetric-GARCH type models have been proposed as suitable models. The symmetric-GARCH models have several shortcomings. For instance, they fail to model the leverage effect in a situation when an unanticipated reduction in prices increases predictable volatility more than an unanticipated increase in prices of similar magnitude. Besides, the symmetric GARCH models do not capture the thick tails property of higher-frequency data. More importantly, the symmetric GARCH models have been criticized since the conditional variance only relies on the magnitude of change, assuming that past positive and negative changes affect the current volatility. Since the conditional variance must be non-negative, the parameters are often constrained to be non-negative, which may not hold when modelling the time series data. As a result, asymmetric-GARCH type models have been favoured since they tolerate asymmetric effects of positive and negative innovation (Hafner & Linton, 2017). Therefore, the current study evaluated how asymmetric-GARCH type models (EGARCH, GJR-GARCH, APARCH, TGARCH, and IGARCH models) fit Kenya’s exchange rate and BoP data.

The study used the available monthly exchange rates data in Kenya from January 1993 to June 2021 as a convenience sample. The sampled data had 342 data points considered adequate for a time series analysis technique. The available monthly balance of payment dataset spanned between August 1998 to June 2021. The study's two secondary data sets were extracted from the Central Bank of Kenya website.

Kenya's exchange rate data trend analysis showed an overall increasing trend. The electioneering period, for instance, around 1992, 2007, and 2012 have been associated with a sharp appreciation due to decreased investor confidence, especially by the foreigner. The long periods of exchange rate stability are seen when there is a smooth government transition. However, the recent COVID-19 has seen a sharp appreciation in Kenya's exchange rates. Regarding BoP, Kenya has been in a deficit over the past decade due to highly valued imported merchandise compared to low valued agricultural exports. Kenya's exports basket primarily comprises low-valued agricultural products, including tea, horticulture, apparel and clothing accessories, coffee, and Tobacco. On the other hand, Kenya's import basket is highly valued finished or intermediate products, including industrial machinery, petroleum products, iron and steel, road motor vehicles, and medicinal or pharmaceutical products.

The comparison of the fitted asymmetric GARCH type models demonstrated that the optimal model for the exchange rates data is APARCH (1,1) - ARMA (3,0) model with a skewed normal distribution. Regarding Kenya's BoP, the optimal model is the ARMA (1,1)-IGARCH (1,1) model with generalized error distribution. Volatility clustering was present in exchange rate data only. The results revealed the volatility persistence with a rapid decrease of the increases in the conditional variance due to shocks. Leverage effect was absent in both series. Regarding BoP, the parametrisation IGARCH imposes the parameters such that the leverage effect parameter is not reported. However, in exchange rate data a significant positive leverage parameter was found. Then, the resultant positive coefficient of λ (positive asymmetry) shows the absence of leverage result in the exchange rate series. The results are inconsistent with the theoretical perspective however, past empirical studies have found absence of the leverage effect.

Estimated Kenya's exchange rate volatility narrows over time, indicating sustained exchange rate stability. However, the results showed that the volatility estimates of BoP keep increasing over time, indicating that the BoP deficit is widening and unstable. The high volatility can be attributed to Kenya's export and import basket composition. Kenya's import basket comprises highly valued finished or intermediate goods, such as electronics, industrial machinery, and motor vehicles, which appreciates over time. On the contrary, the exports encompass low-valued agricultural products such as tea, horticulture, apparel, coffee, and tobacco produced seasonally. Besides, 2012 to 2019 had huge investments in infrastructure investments in roads and railway transport. However, the BoP volatility significantly dropped towards the end of 2019 to mid of 2020. The sharp fall can be attributed to the global COVID-19 pandemic. The pandemic saw a restriction on international merchandise trade.

5.2 Conclusion

The predicted volatilities were captured and are consistent with the shifts in internal and external structural adjustments or shocks. For instance, concerning Exchange rates, volatility was high in the election periods. The recurrent election period in Kenya is a disincentive to investors due to uncertainty on the outcome of the elections. Thus, the electioneering period is linked to higher exchange rate volatility. Operating under the free-float exchange rate, Kenya's exchange rate is susceptible to global economic shocks such as the 2008/09 global financial crisis. The recent COVID-19 has also seen a sudden spike in the exchange rate volatility during 2020. While the exchange rate model will remain stable over the next 12 months, its volatility increases over time.

The model's prediction intervals indicate a diverging uncertainty in exchange rates over the next year, signifying a long-run depreciation trend. Besides, the low order is consistent with the stylized fact that economic or financial series are influenced by recent past shocks rather than distant past shocks. Regarding exchange rate shifts in trade patterns due to the seasonal nature of the agricultural sector plays a crucial role in BoP volatility. Notably, Kenya's main export basket consists of agricultural products including tea, horticulture, articles of apparel and clothing accessories, coffee, and Tobacco which are seasonal in nature. Besides, the vast infrastructural investments in roads and the recent standard railway transport have attracted periodically disbursed loans.

5.3 Recommendation of the Study

i. Due to the relative advantage of asymmetric-GARCH models over symmetric ARCH and GARCH models in capturing volatility, policymakers should adopt them to forecast Kenya's monthly exchange rate and BoPs data adequately using the respective best fit models.

ii. Presence of exchange rate volatility indicates that it is susceptible to future external shocks, which can either weaken or strengthen it. Therefore, investors in the financial market should be cautious when trading.

iii. Declining balance of payment and its volatility indicates that the balance of payment deficit is widening and unstable. Thus, the government should maintain it competitiveness in the global market to attract foreign direct investment and improve net export.

5.4 Suggestions for Further Study

i. Future studies can consider hybrid models in fitting the respective series. For instance, having demonstrated that ARMA (1,1)-IGARCH (1,1) model is the best fit for BoP data, future studies can examine whether the Artificial neural network (ANN) – ARMA (p, q) – IGARCH (p, q) model can best fit the same series. In this method, a researcher can fit the exchange rate data to the ANN model then extract its residuals. The resultant residuals are then fitted using the ARMA (p, q) – IGARCH (p, q) model. Similarly, ANN – ARMA (p, q) – APARCH (p, q) model is suitable for Kenya's exchange rate data.

ii. The generalizability of the study findings to other exchange rates in the money market cannot be guaranteed since the rates used were USD to KES exchange. Thus, future studies can consider other exchange rates.

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APPENDICES

Appendix I: R Codes for Exchange Rates and Balance of Payment

```
remove(list = ls())
#Exchange rate data
Exc.Rate.Data<-read.csv("Kenya Shilling End Period Exchange Rates P. USD.csv")
head(Exc.Rate.Data)
tail(Exc.Rate.Data)
Exch.Rate_KE<-
ts(Exc.Rate.Data$Kenya.Shilling.End.Period.Exchange.Rates.United.States.dollar,
           start = c(1993,1), frequency = 12,)
date<-seq(as.Date("1993-01-31"),by="1
month",length.out=length(Exc.Rate.Data$Year))
length(date)

library(ggplot2)
Exch_plot<-
ggplot(data=NULL,aes(x=date,y=Exc.Rate.Data$Kenya.Shilling.End.Period.Exchang
e.Rates.United.States.dollar))+
  geom_line()+
  labs(title="(a)", caption="", y="Exhange Rate (Ksh/USD)", x="Year",
color=3,size=12)+
  scale_x_date( date_labels = "%b%Y", breaks = "13 months")+
  theme(text = element_text(size=12),
        axis.text.x = element_text(angle=90, hjust=0)) +
  theme(axis.text.x = element_text(angle = 90, vjust=0.5, size = 12))+
  theme(axis.text.y = element_text(angle = 0, vjust=0, size = 12))+
  theme(panel.grid.major = element_blank(), panel.grid.minor = element_blank())+
  theme(panel.background = element_blank())+
  theme(axis.line = element_line(colour = "black"))

#BoP data
BoP.Data<-read.csv("1112310648_Foreign Trade Summary.csv")
head(BoP.Data)
tail(BoP.Data)

BoP.Data.Curr.M.Ksh<-ts(BoP.Data$Trade.balance,
                  start = c(1998,8), frequency = 12)

date2<-seq(as.Date("1998-08-01"),by="1 month",length.out=length(BoP.Data$Year))
length(date2)

library(ggplot2)
theme_set(theme_bw())
BoP_Plot<-ggplot(data=NULL,aes(x=          date2,y=BoP.Data$Trade.balance))+
geom_line()+
  labs(title=" (b)", caption="", y="Trade Balance (Millions of Ksh)", x="Period",
color=3,size=12)+
  scale_x_date( date_labels = "%b%Y", breaks = "13 months")+
  theme(text = element_text(size=12),
```

```
      axis.text.x = element_text(angle=90, hjust=0)) +
  theme(axis.text.x = element_text(angle = 90, vjust=0.5, size = 12))+
  theme(axis.text.y = element_text(angle = 0, vjust=0, size = 12))+
  theme(panel.grid.major = element_blank(), panel.grid.minor = element_blank())+
  theme(panel.background = element_blank())+
  theme(axis.line = element_line(colour = "black"))

#Visualization
library(ggpubr)
ggarrange(Exch_plot, BoP_Plot, ncol =1)

#-----------------------------For descriptive statistics and more graphing---------------------
-------
if(!require(skimr)){install.packages("skimr")}
library(skimr)
library(scales)
library(gridExtra)
#=============================
Exch.Rate_KE%>% skim()
if(!require(moments)){install.packages("moments")}
library(moments)
skewness(BoP.Data.Curr.M.Ksh)
kurtosis(BoP.Data.Curr.M.Ksh)
shapiro.test(BoP.Data.Curr.M.Ksh)

#BoP data ########
BoP.Data.Curr.M.Ksh%>% skim()
skewness(BoP.Data.Curr.M.Ksh)
kurtosis(BoP.Data.Curr.M.Ksh)
shapiro.test(BoP.Data.Curr.M.Ksh)

#------------------------------- Stationary test--------------------------------
library(tseries)
#Exch rate data #######
#at level ######
adf.test(Exch.Rate_KE)
#log diff ######
Exch.Rate_dLog <- diff(log(abs(Exch.Rate_KE)))

adf.test(Exch.Rate_dLog)
#options(digits=6)
adf.test(Exch.Rate_dLog)

##BoP ##########
adf.test(BoP.Data.Curr.M.Ksh) #at level, stationary

BoP_dLog<-diff(log(abs(BoP.Data.Curr.M.Ksh)))
adf.test(BoP_dLog)

#-------------------------------Time Series Plots--------------------------------------------
```

```
par(mfrow=c(2,1))
plot.ts(Exch.Rate_dLog, ylab ="Log difference", main ="(a) Exhange Rate data")
plot.ts(BoP_dLog, ylab="Log difference", main = "(b) BoP data")
par(mfrow=c(1,1))

#-------------------- Descriptive STATS of the d logged series-------------------------------
-------
Exch.Rate_dLog%>% skim
skewness(Exch.Rate_dLog)
kurtosis(Exch.Rate_dLog)
shapiro.test(Exch.Rate_dLog)


BoP_dLog%>% skim()
skewness(BoP_dLog)
kurtosis(BoP_dLog)
shapiro.test(BoP_dLog)



#-------------------------------Histograms of d logs--------------------------------------------
par(mfrow=c(2,1))
hist(Exch.Rate_dLog, ylab ="Frequency", main ="(a) Log-differenced Exhange Rate
data", breaks = 36)
hist(BoP_dLog, ylab="Frequency", main = "(b) Log-differenced BoP data", breaks =
36)
par(mfrow=c(1,1))

#Asymmetric garch ####
#MEAN EQUATION
#STL decomposition ####
decomp = stl(Exch.Rate_dLog, s.window="periodic")
plot(decomp)

library(fpp2)
auto.arima(Exch.Rate_dLog)

auto.arima(Exch.Rate_dLog, trace = TRUE,
        approximation = T,
        stepwise = T,
        seasonal = F, #set to false since the goal is to account for seasonality by
        #incorporating Fourier terms in the model. SARIMA(p, d, q)(P, D, Q)[12]
        #parameters aren't supported by `rugarch`, thus ARMA terms plus adequate
external regressors are needed.
        xreg = fourier(Exch.Rate_dLog, K = 6, h = NULL),
        #the biggest possible K would be 6 (1 TO 6), ALL FAVOURED ARIMA (3,0,0)
        lambda = NULL,
        biasadj = F)
```

```
##testing for ARCH effects #### ---------------------------------------------------------------
----
if(!require("FinTS")){install.packages("FinTS")}
library(FinTS) #for function `ArchTest()`
ArchTest.1 <- ArchTest(Exch.Rate_dLog, lags=12, demean=TRUE)
ArchTest.1

ArchTest.2 <- ArchTest(BOP_dLog, lags=12, demean=TRUE)
ArchTest.2

#Simple ARCH
arch.M<- garch(Exch.Rate_dLog,c(0,1))
summary(arch.M)

###estimated ARCH(1)  variance
hhat <- ts(2*arch.M$fitted.values[-1,1]^2)
plot.ts(hhat)
plot.ts(arch.M$fitted.values[-1,1])
arch.M$fitted.values[-1,1]

#  GARCH(1,1)
if(!require("fGarch")){install.packages("fGarch")}

library(fGarch)
GARCH_Model <- garchFit(data = Exch.Rate_dLog, trace = F)
GARCH_Model

#EGARCH#####
if(!require("rugarch")){install.packages("rugarch")}
require(rugarch)

# Nelson's egarch model #####
egarch11.spec.1 = ugarchspec(variance.model=list(model="eGARCH",
                                     garchOrder=c(1,1)),
                      mean.model=list(armaOrder=c(3,0)),
                      distribution.model="snorm"#Others:      sstd",std,       "ged","norm"
#SKEWED NORMAL PREFERRED
                      ,fixed.pars=list(omega= 1.999e-05)
)
Exch.egarch11.fit.1 = ugarchfit(egarch11.spec.1, Exch.Rate_dLog)
Exch.egarch11.fit.1
# Estimated standardized returns
stdret <- residuals(Exch.egarch11.fit.1, standardize = F)
mean(stdret)
mean(abs(stdret))
##rmae
sqrt(mean(abs(stdret)))

#IGARCH model #####
igarch11.1.spec.1 = ugarchspec(variance.model=list(model="iGARCH",
```

```
                    garchOrder=c(1,1)),
            mean.model=list(armaOrder=c(3,0),include.mean=TRUE),
            distribution.model="snorm", #Others: sstd",std, ged
            fixed.pars=list(omega= 1.999e-05))#delta =1

Exch.igarch11.1.fit.1 = ugarchfit(igarch11.1.spec.1, Exch.Rate_dLog)
Exch.igarch11.1.fit.1

# Estimated standardized returns
stdret <- residuals(Exch.igarch11.1.fit.1, standardize = F)
mean(stdret)
#rmae
sqrt(mean(abs(stdret)))



#APAGARCH #########
apagarch11.1.spec.1 =
ugarchspec(variance.model=list(model="fGARCH",garchOrder=c(1,1),submodel =
"APARCH"),
             mean.model=list(armaOrder=c(3,0),include.mean=TRUE),
             distribution.model="snorm", #fixed.pars=list(omega=0))
             fixed.pars=list(omega= 1.999e-05))


# bEST #########
Exch.apagarch11.1.fit.1 = ugarchfit(apagarch11.1.spec.1, Exch.Rate_dLog)
Exch.apagarch11.1.fit.1
# Estimated standardized returns
stdret <- residuals(Exch.apagarch11.1.fit.1, standardize = F)
mean(stdret)
#rmae
sqrt(mean(abs(stdret)))


# TGARCH model #####
Tgarch11.1.spec =
ugarchspec(variance.model=list(model="fGARCH",garchOrder=c(1,1),submodel =
"TGARCH"),
            mean.model=list(armaOrder=c(3,0),
                  include.mean=TRUE),
            distribution.model="snorm", #fixed.pars=list(omega=0))
            fixed.pars=list(omega= 1.999e-05))

Exch.Tgarch11.1.fit = ugarchfit(Tgarch11.1.spec, Exch.Rate_dLog)
Exch.Tgarch11.1.fit
# Estimated standardized returns
stdret <- residuals(Exch.Tgarch11.1.fit, standardize = F)
mean(stdret)
mean(abs(stdret))
```

```
#rmae
sqrt(mean(abs(stdret)))

# GJR-GARCH model ####
gjrgarch11.spec = ugarchspec(variance.model=list(model="gjrGARCH",
                                                  garchOrder=c(1,1)),
                              mean.model=list(armaOrder=c(3,0),include.mean=TRUE),
                              distribution.model="snorm",
                              fixed.pars=list(omega= 1.999e-05))
Exch.gjrgarch11.fit = ugarchfit(gjrgarch11.spec, Exch.Rate_dLog)
Exch.gjrgarch11.fit

# Estimated standardized returns
stdret <- residuals(Exch.gjrgarch11.fit, standardize = F)
mean(stdret)
#rmae
sqrt(mean(abs(stdret)))


##Diagnostics ####

#BEST FIT ####
Exch.apagarch11.1.fit.1

# Using the method sigma to retrieve the estimated Volailities
garchvol <- sigma(Exch.apagarch11.1.fit.1)

# Plot the volatility for 2017
plot(garchvol["2013"])

plot(garchvol, main ="Estimated Volatilities")
plot(Exch.apagarch11.1.fit.1, which=9)
class(Exch.apagarch11.1.fit.1)
#

plot(Exch.apagarch11.1.fit.1, which =2)
# series with 95% conf. int (+/- 2 conditional std. dev.)

plot(Exch.apagarch11.1.fit.1,which = 2) #TO 12
plot(Exch.apagarch11.1.fit.1, which = 4)

plot(Exch.apagarch11.1.fit.1, which = 10)  #ACF of residual


plot(residuals(Exch.apagarch11.1.fit.1, standardize = TRUE), type = "h",
    main = "Residuals from ARMA(3, 0) - APARCH (1, 1) model\n Data: Kenya's
Exchange Rates",
    major.ticks = "years", grid.ticks.on = "years")
plot(Exch.apagarch11.1.fit.1, which =8)
```

```
par(mfrow=c(1,2))

plot(residuals(Exch.apagarch11.1.fit.1, standardize = TRUE), type = "h",
    main = "Residuals from ARMA(3, 0) - APARCH (1, 1) model\n",
    major.ticks = "years", grid.ticks.on = "years")
plot(Exch.apagarch11.1.fit.1, which =8)
par(mfrow=c(1,1))

# Compute unconditional volatility
sqrt(uncvariance(Exch.apagarch11.1.fit.1))


# Forecast volatility 12 months ahead
garchforecast <- ugarchforecast(fitORspec = Exch.apagarch11.1.fit.1,
                              n.ahead = 12)

plot(garchforecast,which = 1)

par(mfrow=c(1,2))
plot(Exch.apagarch11.1.fit.1, which =2, ylab= "Series")
plot(garchforecast,which = 1)
par(mfrow=c(1,1))


garchforecast
# Extract the predicted volatilities and print them
print(sigma(garchforecast))
print(fitted(garchforecast))


###BoP Modelling ###########

#MEAN EQUATION
#STL decomposition ####
decomp = stl(BoP_dLog, s.window="periodic")
plot(decomp)

auto.arima(BoP_dLog)
#ARIMA(3,0,2)(1,0,0)[12] with non-zero mean

#fourier pars
auto.arima(BoP_dLog, trace = TRUE,
        approximation = T,
        stepwise = T,
        seasonal = F, #set to false since the goal is to account for seasonality by
        #incorporating Fourier terms in the model. SARIMA(p, d, q)(P, D, Q)[12]
        #parameters aren't supported by `rugarch`, thus ARMA terms plus adequate
external regressors are needed.
        xreg = fourier(BOP_dLog, K = 6, h = NULL),
```

```
        #the frequency of the ts is 12 so that the biggest possible K would be 6 (1 TO 6),
ALL FAVOURED ARIMA (3,0,0)
        lambda = NULL,
        biasadj = F)



GARCH_Model_2 <- garchFit(data = BoP_dLog, trace = F)
GARCH_Model_2
#  mu      omega     alpha1      beta1
#0.0096173  0.0074497  0.2900864  0.6475133

#EGARCH#####
# Nelson's egarch model ----------------------------------------------------

egarch11.spec.2 = ugarchspec(variance.model=list(model="eGARCH",
                                    garchOrder=c(1,1)),
                        mean.model=list(armaOrder=c(0,3)),
                        distribution.model="ged"#Others:       sstd",std,        "ged","norm"
#SKEWED NORMAL PREFERRED
                        ,fixed.pars=list(mu = 0.0096173)
)
BoP.egarch11.fit.2 = ugarchfit(egarch11.spec.2, BoP_dLog)
BoP.egarch11.fit.2

# Estimated standardized returns
stdret <- residuals(BoP.egarch11.fit.2, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))
##2 -------------------------
egarch11.spec.2 = ugarchspec(variance.model=list(model="eGARCH",
                                    garchOrder=c(1,1)),
                        mean.model=list(armaOrder=c(1,1)),
                        distribution.model="ged"#Others:       sstd",std,        "ged","norm"
#SKEWED NORMAL PREFERRED
                        ,fixed.pars=list(mu = 0.0096173)
)
BoP.egarch11.fit.2 = ugarchfit(egarch11.spec.2, BoP_dLog)
BoP.egarch11.fit.2
# Estimated standardized returns
stdret <- residuals(BoP.egarch11.fit.2, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))
#IGARCH model ##### --------------------------------------------------------------------------
-----
igarch11.1.spec.1 = ugarchspec(variance.model=list(model="iGARCH",
```

```
                    garchOrder=c(1,1)),
          mean.model=list(armaOrder=c(0,3),include.mean=TRUE),
          distribution.model="ged", #Others: sstd",std, ged
          fixed.pars=list(mu = 0.0096173))

BoP.igarch11.1.fit.1 = ugarchfit(igarch11.1.spec.1, BoP_dLog)
BoP.igarch11.1.fit.1

# Estimated standardized returns
stdret <- residuals(BoP.igarch11.1.fit.1, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))

##2--------------------------
igarch11.1.spec = ugarchspec(variance.model=list(model="iGARCH",
                    garchOrder=c(1,1)),
          mean.model=list(armaOrder=c(1,1),include.mean=TRUE),
          distribution.model="snorm", #Others: sstd",std, ged
          fixed.pars=list(mu = 0.0096173))#delta =1

BoP.igarch11.1.fit = ugarchfit(igarch11.1.spec, BoP_dLog)
BoP.igarch11.1.fit

# Estimated standardized returns
stdret <- residuals(BoP.igarch11.1.fit, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))

#APAGARCH ######### ---------------------------------------------------------------------
-------
apagarch11.1.spec.1 =
ugarchspec(variance.model=list(model="fGARCH",garchOrder=c(1,1),submodel =
"APARCH"),
          mean.model=list(armaOrder=c(0,3),include.mean=TRUE),
          distribution.model="snorm", #fixed.pars=list(omega=0))
          fixed.pars=list(mu = 0.0096173))

BoP.apagarch11.1.fit.1 = ugarchfit(apagarch11.1.spec.1, BoP_dLog)
BoP.apagarch11.1.fit.1
# Estimated standardized returns
stdret <- residuals(BoP.apagarch11.1.fit.1, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))
##2--------------------------
```

```
apagarch11.1.spec =
ugarchspec(variance.model=list(model="fGARCH",garchOrder=c(1,1),submodel =
"APARCH"),
                mean.model=list(armaOrder=c(1,1),include.mean=TRUE),
                distribution.model="snorm", #fixed.pars=list(omega=0))
                fixed.pars=list(mu = 0.0096173))

BoP.apagarch11.1.fit = ugarchfit(apagarch11.1.spec, BoP_dLog)
BoP.apagarch11.1.fit
# Estimated standardized returns
stdret <- residuals(BoP.apagarch11.1.fit, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))


# TGARCH model #####-----------------------------------------------------------------------
-
Tgarch11.1.spec =
ugarchspec(variance.model=list(model="fGARCH",garchOrder=c(1,1),submodel =
"TGARCH"),
               mean.model=list(armaOrder=c(0,3),
                       include.mean=TRUE),
               distribution.model="snorm", #fixed.pars=list(omega=0))
               fixed.pars=list(mu = 0.0096173))

BoP.Tgarch11.1.fit = ugarchfit(Tgarch11.1.spec, BoP_dLog)
BoP.Tgarch11.1.fit
# Estimated standardized returns
stdret <- residuals(BoP.Tgarch11.1.fit, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))

##2--------------------------
Tgarch11.1.spec.1 =
ugarchspec(variance.model=list(model="fGARCH",garchOrder=c(1,1),submodel =
"TGARCH"),
                mean.model=list(armaOrder=c(1,1),
                        include.mean=TRUE),
                distribution.model="snorm", #fixed.pars=list(omega=0))
                fixed.pars=list(mu = 0.0096173))

BoP.Tgarch11.1.fit.2 = ugarchfit(Tgarch11.1.spec.1, BoP_dLog)
BoP.Tgarch11.1.fit.2
# Estimated standardized returns
stdret <- residuals(BoP.Tgarch11.1.fit.2, standardize = F)
mean(stdret)
```

```
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))


# GJR-GARCH model ######------------------------------------------------
gjrgarch11.spec = ugarchspec(variance.model=list(model="gjrGARCH",
                                          garchOrder=c(1,1)),
                        mean.model=list(armaOrder=c(0,3),include.mean=TRUE),
                        distribution.model="snorm",
                        fixed.pars=list(mu = 0.0096173))
BoP.gjrgarch11.fit.1 = ugarchfit(gjrgarch11.spec, BoP_dLog)
BoP.gjrgarch11.fit.1

# Estimated standardized returns
stdret <- residuals(BoP.gjrgarch11.fit.1, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))

##2--------------------------
gjrgarch11.spec = ugarchspec(variance.model=list(model="gjrGARCH",
                                          garchOrder=c(1,1)),
                        mean.model=list(armaOrder=c(1,1),include.mean=TRUE),
                        distribution.model="snorm",
                        fixed.pars=list(mu = 0.0096173))
BoP.gjrgarch11.fit.2 = ugarchfit(gjrgarch11.spec, BoP_dLog)
BoP.gjrgarch11.fit.2

# Estimated standardized returns
stdret <- residuals(BoP.gjrgarch11.fit.2, standardize = F)
mean(stdret)
mean(abs(stdret))
#rmae
sqrt(mean(abs(stdret)))

##Diagnostics

#BEST FIT #####
BoP.igarch11.1.fit

# Using the method sigma to retrieve the estimated Volatilities
garchvol <- sigma(BoP.igarch11.1.fit)

# Plot the volatility for 2017
plot(garchvol["2013"])

plot(garchvol, main ="Estimated Volatilities")
plot(BoP.igarch11.1.fit, which=9)
```

```
class(BoP.igarch11.1.fit)
#

plot(BoP.igarch11.1.fit, which =2)
# series with 95% conf. int (+/- 2 conditional std. dev.)

plot(BoP.igarch11.1.fit,which = 2) #TO 12
plot(BoP.igarch11.1.fit, which = 4)

plot(BoP.igarch11.1.fit, which = 10)  #ACF of residual

plot(residuals(BoP.igarch11.1.fit, standardize = TRUE), type = "h",
    main = "Residuals from ARMA(1, 1) - IGARCH (1, 1) model\n Data: Log-differenced sereis of Kenya's BoP",
    major.ticks = "years", grid.ticks.on = "years")
plot(BoP.igarch11.1.fit, which =8)

par(mfrow=c(1,2))
plot(residuals(BoP.igarch11.1.fit, standardize = TRUE), type = "h",
    main = "Residuals from ARMA(1, 1) - IGARCH (1, 1) model\n Data: Log-differenced sereis of Kenya's BoP",
    major.ticks = "years", grid.ticks.on = "years")
plot(BoP.igarch11.1.fit, which =8)
par(mfrow=c(1,1))

# Compute unconditional volatility
sqrt(uncvariance(BoP.igarch11.1.fit))

# Forecast volatility 12 months ahead
garchforecast <- ugarchforecast(fitORspec = BoP.igarch11.1.fit,
                          n.ahead = 12)

plot(garchforecast,which = 1)
par(mfrow=c(1,2))
plot(BoP.igarch11.1.fit, which =2, ylab= "Series")
plot(garchforecast,which = 1)
par(mfrow=c(1,1))

garchforecast
# Extract the predicted volatilities and print them
print(sigma(garchforecast))
print(fitted(garchforecast))

plot(BoP.Data.Curr.M.Ksh)
lines(fitted(garchforecast), col =4)
##
```

Appendix II: Chuka University Ethics Committee Letter

CHUKA UNIVERSITY

Knowledge is Wealth *(Sapientia divitia est)* Akili ni Mali

CHUKA UNIVERSITY INSTITUTIONAL ETHICS REVIEW COMMITTEE

Telephones: 020-2310512/18 **P. O. Box 109-60400, Chuka**

Direct Line: 0772894438 **Email: info@chuka.ac.ke,**

Website: www.chuka.ac.ke

REF: CUIERC/ NACOSTI/ 164 **30/ July/2021**

TO; **Eric Munene Ndege**

RE; **Application of Asymmetric –Garch Type Models to The Kenyan Exchange Rate and Balance of Payments**

This is to inform you that *Chuka University IERC* has reviewed and approved your above research proposal. Your application approval number is *NACOSTI/NBC/AC-0812.* The approval period is 30/July/ 2021 -30/July 2022

This approval is subject to compliance with the following requirements;

i. Only approved documents including (informed consents, study instruments, MTA) will be used

ii. All changes including (amendments, deviations, and violations) are submitted for review and approval by *Chuka University IERC.*

iii. Death and life threatening problems and serious adverse events or unexpected adverse events whether related or unrelated to the study must be reported to *Chuka University IERC* within 72 hours of notification

iv. Any changes, anticipated or otherwise that may increase the risks or affected safety or welfare of study participants and others or affect the integrity of the research must be reported to *Chuka University IERC* within 72 hours

v. Clearance for export of biological specimens must be obtained from relevant institutions.

vi. Submission of a request for renewal of approval at least 60 days prior to expiry of the approval period. Attach a comprehensive progress report to support the renewal.

vii. Submission of an executive summary report within 90 days upon completion of the study to *Chuka University IERC.*

Prior to commencing your study, you will be expected to obtain a research license from National Commission for Science, Technology and Innovation (NACOSTI) https://oris.nacosti.go.ke and also obtain other clearances needed.

Yours sincerely

for PROF.ADIEL MAGANA

CHAIRMAN CHUKA UNIVERSITY

Appendix III: NACOSTI Permit

REPUBLIC OF KENYA

NATIONAL COMMISSION FOR SCIENCE,TECHNOLOGY & INNOVATION

Ref No: **519602**

Date of Issue: **14/October/2021**

RESEARCH LICENSE

This is to Certify that Mr.. ERIC NDEGE MUNENE of Chuka University, has been licensed to conduct research in Tharaka-Nithi on the topic: APPLICATION OF ASYMMETRIC-GARCH TYPE MODELS TO THE KENYAN EXCHANGE RATE AND BALANCE OF PAYMENTS for the period ending : 14/October/2022.

License No: **NACOSTI/P/21/13277**

519602

Applicant Identification Number

Director General

NATIONAL COMMISSION FOR SCIENCE,TECHNOLOGY & INNOVATION

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