CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE(MATHEMATICS), BACHELORS OF SCIENCE, BACHELORS OF ARTS(MATHS-ECONS), BACHELORS OF SCIENCE(ECON STATS)

MATH 301: LINEAR ALGEBRA II

STREAMS: AS ABOVE **TIME: 2 HOURS**

DAY/DATE: FRIDAY 14/12/2018 **8.30 A.M - 10.30 A.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: [30 MARKS]

a) Find the symmetric matrix that correspond to the following quadratic form

$$q(x,y,z)=2x^2-5xy+3y^2-16xz+14yz-6z^2$$

[3 Marks]

b) Prove that similar matrices have the same characteristic polynomial. [3 Marks]

c) Show that if $A=\begin{bmatrix}1 & -1\\ 3 & 4\end{bmatrix}$, then A is a zero of the function $f(t)=t^2-5t+7$ [3 Marks]

d) Given that $A=\begin{bmatrix}2 & -3 & 3\\ 3 & -4 & 3\\ 6 & -6 & 5\end{bmatrix}$, find the eigenvalues of A^{-1} [5 Marks]

e) Apply Gram-Schmidtorthogonalization process to the basis $B = \{1, x, x^2\}$ in $P_2(x)$ with the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$ to obtain an orthogonal set [5 Marks]

f) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$ [5 Marks]

g) Let $B = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 3 & 4 \\ 7 & -5 & 2 \end{bmatrix}$

i. Show that B is not an orthogonal matrix [3 Marks]

ii. Find an orthonormal matrix relative to B [3 Marks]

QUESTION TWO: [20 MARKS]

a) Let f be a bilinear form on R^2 defined by $f[(x_1, x_2), (y_1, y_2)] = x_1 y_1 - 3x_1 y_2 + 2x_2 y_2$. Find

i. The matrix A of f in the basis $\{u_1 = (1,0), u_2 = (0,1)\}$

ii. The matrix B of f in the basis $\{v_1 = (2,1), v_2 = (1,-1)\}$

iii. The change of basis matrix P from the basis $\{u_i\}$ to the basis $\{v_i\}$ and verify that $B = P^T AP$. [12 Marks]

b) Given that $A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 5 & -3 & 1 \\ 1 & 6 & -2 & 4 \\ 0 & 1 & -3 & 7 \end{bmatrix}$ determine the number n_k and the sum S_k of principal minors of order 1, 2 and 4. [8 Marks]

QUESTION THREE: [20 MARKS]

a) Let A be the matrix

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -5 \\ -3 & -5 & 8 \end{bmatrix}$$

Apply diagonalization algorithm to obtain a matrix P such that $D = P^T AP$ [6 Marks]

b) Let $A = \begin{bmatrix} 3 & -1 & 1 \\ 7 & -5 & 1 \\ 6 & -6 & 2 \end{bmatrix}$

i. Find the characteristic polynomial of A. [4 Marks]

ii. Find all the eigenvalues of A and their corresponding eigenvectors. [5 Marks]

iii. Is A diagonalizable? Verify [2 Marks]

iv. Determine the algebraic and geometric multiplicities of each of the eigenvalues of A [3 Marks]

QUESTION FOUR: [20 MARKS]

a) State Cayley-Hamilton theorem and verify using a linear operator $T : R^2 \to R^2$ defined by $T(x, y,) = (2x - 3y, x + 5y)$ [6 Marks]

b) Find the characteristic polynomial and hence the minimal polynomial of the matrix

A= $\begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$ [6 Marks]

c) Prove that if v_1 and v_2 are eigenvectors of a matrix A corresponding to distinct eigenvalues, then v_1 and v_2 are linearly independent. [5 Marks]

d) Let A be an nxn matrix over a field K. show that the mapping $f(X,Y) = X^T AY$ is a bilinear form on K^n [3 Marks]

QUESTION FIVE: [20 MARKS]

a) Show that the function $f: R^2 \to R$ defined by $f(u,v) = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$ where $u = (x_1, x_2)$ and $v = (y_1, y_2)$ defines an inner product in R^2. [5 Marks]

b) Determine an orthogonal matrix P whose first row is $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$ [5 Marks]

c) Evaluate the matrix Q that represent the usual inner product in C^3 relative to the basis $\{i, i+1, i-1\}$ and show that's its hermitian. [5 Marks]

d) Prove an orthogonal set of n vectors is linearly independent. [5 Marks]

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