CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, ART AND EDUCATION

MATH 241: PROBABILITY AND STATISTICS I

STREAMS: BSC,BED,BA **TIME: 2 HOURS**

DAY/DATE: THURSDAY 13/12/2018 **2.30 P.M – 4.30 P.M**

INSTRUCTIONS:

- Answer Question ONE and any other TWO Questions.

QUESTION ONE [30 MARKS]

a) A random variable Y has cumulative distribution function given by:

$$F(y)=\begin{cases} 0, & y<0 \\ \frac{13}{27}, & 0\le y<3 \\ \frac{72}{81}, & 3\le y<8 \\ 1, & y\ge 8 \end{cases}$$

i) Determine the p.d.f of Y

ii) Compute $p(1\le y\le 5)$

(5 marks)

b) If the moment generating function of a random variable X is given by $M(t)=(2-5t)^{-6}$

Determine:

i) Mean of X (2 marks)

ii) Variance of X (3 marks)

iii) $E[(X+4)^2]$ (2 marks)

c) A fair coin is tossed 256 times. Using normal approximation to binomial probabilities, determine the probability of obtaining:

i) At least 115 heads

ii) Between 113 and 145 heads
(5 marks)

d) The time taken for a car engine to cool as observed by a vehicle dealers firm has a distribution measured in hours given by

$$f(y)=\begin{cases}\frac{k}{10}(y-9), 0\le y<10\\ 0, elsewhere\end{cases}$$

i) Find the value of k that makes the above distribution a varied p.d.f (2 marks)

ii) Find the median time.
(3 marks)

iii) Find the variance of Y
(3 marks)

e) Let X be a continuous random variable with p.d.f given by

$$f(x)=\begin{cases}\frac{1}{8}(x+1), 2\le x<4\\ 0, elsewhere\end{cases}$$

Given that $Y=2x+1,$ find the

i) Probability density function of Y, [g(y)] and
ii) The cumulative distribution of Y, [G(y)]
(5 marks)

QUESTION TWO (20 Marks)

a) Let Y be a random variable with probability density function

$$f(y)=\begin{cases}\frac{3}{64}y^2(4-y), 0\le y<4\\ 0, otherwise\end{cases}$$

i) Verify that $f(y)$ is a probability distribution of the random variable Y for the given values.
ii) Find the first, second and third central moments of Y and hence its variance
iii) Find the mode of Y. (16 marks)

b) Let the variable X have the distribution $P(X=0)=P(X=2)=p$, $P(X=1)=1-2p$, for $0\le p\le\frac{1}{2}$. For what value of p is the variance of X maximum? (4 marks)

QUESTION THREE (20 Marks)

a) A discrete random variable Y has a probability mass function given by

$$f(y)=\begin{cases}\left(\frac{1}{4}\right)^{y}\left(\frac{3}{4}\right), y=0,1,2,\ldots \\ 0, otherwise\end{cases}$$

i) Determine the factorial moment generating function of Y,
ii) Use the f.m.g.f in (i) above to compute the mean and variance of Y.
iii) Hence compute the first four probabilities.
(10 marks)

b) A geometric random variable X with parameter δ has the probability distribution given as

$$f(x)=\begin{cases}\delta(1-\delta)^{x-1}, x=1,2,\ldots \\ 0, otherwise\end{cases}$$

i) Obtain the moment generating function of X.
(5 marks)
ii) Use the m.g.f obtained in (i) above to find the mean and variance of X. (5 marks)

QUESTION FOUR (20 Marks)

a) A random variable X has a probability density function below

$$f(x)=\begin{cases}ax^2+b, 0\le x<1 \\ 0, otherwise\end{cases}$$

i) Given that $E(X)=\frac{2}{3}$, determine the values of a and b hence the standard deviation of X.
(10 marks)
ii) Find $E(2x+3)^2$
(3 marks)

b) On the basis of a part time experience, a car sales girl knows that the number of cars she sells per week is a random variable X with probability mass function below

X	0	1	2	3	4	5	6	7	8
$p(x)$	0.1	3m	0.25	0.15	m	0.04	0.03	0.02	0.01

i) Find the values of m
ii) Find the mean number of cars sold per week
iii) Calculate the variance of X
(7 marks)

QUESTION FIVE (20 Marks)

a) The average length of super loaf bread distributed to local stores by a certain bakery is 30 cm and the standard deviation is 2 cm. Assuming the length is normally distributed, what is the probability of the loaf being:

i) Longer than 32.5 cm

ii) Between 28.9 cm and 32.5 cm

iii) Shorter than 26.5 (6 marks)

iv) The value of r such that $P[X<r]=0.8686$ (3 marks)

v) Determine the number of loaves with length less than 27.8 cm in a crate of 25 loaves. (3 marks)

b) Bulbs are manufactured by a machine and it is known that approximately 25% are outside certain tolerance limits. If a random sample of 450 bulbs is taken, find the probability that more than 75 bulbs will be outside the tolerance limits. (3 marks)

c) Let F(x) be the C.D.F of a poisson distribution with parameter λ. If F(2)=2F(1).
Find λ.
(5 marks)
