CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE /ARTS; BSc. MATHEMATICS; ECONOMICS AND STATISTICS; BACHELORS OF ARTS (MATHS-ECONS)

MATH 222: VECTOR ANALYSIS

STREAMS: AS ABOVE **TIME: 2 HOURS**

DAY/DATE: FRIDAY 14/12/2018 **8.30 P.M - 4.30 P.M.**

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: [30 MARKS]

a) Prove that if $\vec{a}$ and $\vec{b}$ are non-collinear vectors and $x_1\vec{a}+y_1\vec{b}=x_2\vec{a}+y_2\vec{b}$, then $x_1=x_2$ and $y_1=y_2$ [2 Marks]

b) Find the directional derivative of $\phi = x^2y + xz$ in the direction of the vector $2i-2j+k$ at (1,-2,2) [4 Marks]

c) Calculate the area of the parallelogram of PQRS, where P(1,1), Q(2,3), R(5,4) and S(4,2). [4 Marks]

d) Evaluate the line integral $\int_c ydx + zdy + xdz$ where c is the parametric curve $x = t, y = t^2, x = t^3; 0 \le t \le 1$

[4 Marks]

e) Determine the unit tangent vector at the point where $t=2$ on the curve $x=t^2+1, y=4t-3, z=2t^2-6t$

[3 Marks]

f) Given that $\phi = \ln|r|$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|=\sqrt{x^2+y^2+z^2}$. Find $div(grad\phi)$

[5 Marks]

g) A curve C is defined by parametric equations $x = x(s)$, $y = y(s)$ and $z = z(s)$ where s is the length of the curve C measured from a fixed point C. If $\vec{r}$ is the position vector of any point on C, show that $\frac{d\vec{r}}{ds}$ is a unit vector tangent to C. [4 Marks]

h) State without proof the following theorems
   i. Green's theorem in a plane [2 Marks]
   ii. Stoke's theorem [2 Marks]

QUESTION TWO: [20 MARKS]

a) Find the equation for the plane perpendicular to the vector $\vec{A}=2\hat{i}-3\hat{j}+6\hat{k}$ and passing through the terminal point of the vector B ¿$2\hat{i}-5\hat{j}+3\hat{k}$ [4 Marks]

b) Prove that $div(curl(\overrightarrow{A})) = 0$ [5 Marks]

c) A particle moves in space so that its position vector is given by $\vec{r} = \cos wt\ \vec{i} + \sin wt\ \vec{j}$ where w is a constant. Show that ;

i. The velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$ [3 Marks]

ii. The acceleration $\vec{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin [2 Marks]

d) Given $\phi(x,y,z) = 2x^3y^2z^4$, calculate;

i. *Div (grad* ϕ) [3 Marks]

ii. C*url(grad(* ϕ *))* [3 Marks]

QUESTION THREE: [20 MARKS]

a) Given the vector function $\vec{F} = (y^2\cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k$

i. Show that a conservative force field [3 Marks]

ii. Find the scalar potential [4 Marks]

iii. Evaluate $\int_c \vec{F}.\vec{dr}$ where c is a straight line from $(0,-1,1)$ to $(\frac{\pi}{2},-1,2)$ [3 Marks]

b) State athe Frenet-Serret formulas [3 Marks]

c) Given the space curve $r(t) = \langle t, 3\sin t, 3\cos t\rangle$, find:

i. The tangent vector $\hat{T}$ [2 Marks]

ii. The principal normal $\hat{N}$ and curvature κ [2 Marks]

iii. The unit binormal $\hat{B}$ [3 Marks]

QUESTION FOUR: [20 MARKS]

a) Evaluate the line integral $\int_c y^3 dx - x^3 dy$ where C is the positively oriented circle of radius 2 centered at the origin [10 Marks]

b) Use Stoke's theorem to evaluate $\iint_S curl \vec{F} \vec{ds}$ where $\vec{F} = z^2 i - 3xyj + x^3 y^3 k$ and S is the part of the surface $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume S is oriented upwards [10 Marks]

QUESTION FIVE: [20 MARKS]

a) Find the volume of the parallelepiped with adjacent sides

$\vec{u} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{v} = -\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$ [4 Marks]

b) If $\vec{A} = x^2 yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$ and $\vec{B} = 2z\hat{i} + y\hat{j} - x^2\hat{k}$, find $\frac{\partial^2}{\partial_x \partial_y} (\vec{A} X \vec{B})$ at (1,0,-2)

[4 Marks]

Verify divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $0 \le z \le 3$ [12 Marks]

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