CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECONDARY YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

MATH 205: ELEMENTS OF SET THEORY

STREAMS: BSC (MATHS) **TIME: 2 HOURS**

DAY/DATE: FRIDAY 07/12/2018 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) For each of the following cases, determine whether it represents a function. If it is a function, state whether it is injective

i. To each of the 24 student in math 205, assign the gender
ii. To each student in Chuka university, assign a registration number
iii. To each student in first year, assign the semester course units
iv. To each book written by a single author, assign the author
v. To each positive number, assign its square root (5 marks)

b) Given the sets $A_n = \{n, n+2\}$, where n is a positive integer evaluate

i. $\bigcup_{n=3}^{10} A_n$ and $\bigcap_{n=1}^{10} A_n$

ii. $\cup_n A_n$ and $\cap_n A_n$ (4 marks)

c) Find the domain of the function $f : R \to R$ defined by $f(x) = \dfrac{4}{\sqrt{x^2 - 4}}$ (2 marks)

d) Consider the set $A = \left\{ 11 + (-1)^n \frac{1}{n} \right\}$ where n is a positive integer

i. Find the supremum and the infimum of A (2 marks)

ii. Find all the limit points of A (1 marks)

e) With an appropriate example, show that a bounded sequence is not necessarily convergent (2 marks)

f) Consider the function $f : R \to R$ defined by $f(x) = x^2 + 1$

i. Find the largest set D such that $f : D \to R$ is injective

ii. Find the smallest set T such that $f : R \to T$ is onto (3 marks)

g) Let A and B be sets. Show that the product order on $A \times B$ defined by $(a,b) \prec (c,d)$ if $a \le c$ and $b \le d$ is a partial order on $A \times B$ (4 marks)

h) Prove that the set of integers is countable (3marks)

i) Prove that if the limit of a sequence exists, then it is unique (3marks)

j) State the Axiom of choice (1 marks)

QUESTION TWO (20 MARKS)

a) Distinguish the following

i. A restriction and an inclusion map (3 marks)

ii. A countable and uncountable set (2 marks)

iii. A linearly ordered set and a poset (3 marks)

b) Consider the function $f : R \to R$ defined by $f(x) = \frac{|x|}{x} : x \ne 0$ and $f(0) = 0.$ Determine

i. The quotient sets $\frac{R}{f}$ (2 marks)

ii. The image $f(R)$ (2 marks)

c) Given a sequence intervals $I_1, I_2, ...$ such that $I_1 \supseteq I_2 \supseteq ....$, it is called a 'nested' sequence.

i. Give an example of a nested sequenced of nonempty open intervals whose intersection is empty. (2 marks)

ii. Give an example of a nested sequenced of open intervals whose intersection is not empty (2 marks)

iii. Prove that intersection that a nested sequence of closed intervals of the form $I_1 \supseteq I_2 \supseteq ....$ is not empty (4 marks)

QUESTION THREE (20 MARKS)

a) Let A and B be sets in a universal set U, prove that $\chi_{A\cap B} = \chi_A \chi_B$ where χ_A is the characteristic function of A and $\chi_A \chi_B$ is the product of functions (6 marks)

b) Prove the distributive laws i.e.

i. $B \cap (\cup_k A_k) = \cup_k (B \cap A_k)$ (4 marks)

ii. $B \cup (\cap_k A_k) = \cap_k (B \cup A_k)$ (4 marks)

c) Let $A_m = \{m, 2m, 3m, ...: m \in N\}$, determine and explain the following sets

i. $A_3 \cap A_7$ (2 marks)

ii. $A_3 \cup A_7$ (2 marks)

iii. $\cup_m A_m$ (1 marks)

iv. $\cap_m A_m$ (1 marks)

QUESTION FOUR (20 MARKS)

a) The prerequisites in a college is a familiar partial ordering of available classes. Let M be a set of mathematics courses at XYZ College. Define $A \prec B$ if class A is a prerequisite of class B, below is a list of mathematics courses and their prerequisites

Class	Prerequisite
Math 122	None
Math 201	Math 122
Math 205	Math 122
Math 206	Math 205
Math 301	Math 201
Math 302	Math 301

Math 401	Math 201, Math 205
Math 403	math 206, Math401

Required:

i. Draw an Hasse diagram for the partial ordering of these classes (3 marks)
ii. Find all the minimal and maximal elements of these classes (3 marks)
iii. Determine the first and last element if they exist. (2 marks)

A student wants to take eight mathematics courses, but only one per semester;

iv. Which choices does he have in her first and last semester? (2 marks)
v. Suppose he wants to take Math 205 in his first year (first or second semester) and Math 301 in his senior year (7^{th} or 8^{th} semester), explain all possible ways he can take the eight Subjects. (5 marks)

b) Let λ be an ordinal number. Prove that $\lambda+1$ is the immediate successor of λ (5 marks)

QUESTION FIVE (20 MARKS)

a) Prove that the intervals [0,1] and (0,1] are equivalent. (5 marks)
b) Prove that the unit interval [0,1] is non-denumerable (5 marks)
c) Prove that a countable union of finite sets is countable (5 marks)
d) Prove that every infinite set contains a countable set (5 marks)
